This supplement contains a proof of Proposition 1 and additional discussion of Proposition 2 in the published article.

Proof of Proposition 1

Proposition 1. For $0 \leq \beta \leq 1$ and for all $0 \leq \gamma < 1$, $x^P > x^C$.

Proof. Recall that $x^P = \frac{A}{\Phi}(2 - (1 + \beta)\gamma)$, where $\Phi = 4(1 - \gamma)(1 + \gamma)^2 b\delta - (1 + \beta)(2 - (1 + \beta)\gamma) > 0$ is assured by stability conditions, and $x^C = \frac{2A}{\theta}(2 - \beta\gamma)$, where $\theta = (2 - \gamma)(2 + \gamma)^2 b\delta - 2(1 + \beta)(2 - \beta\gamma) > 0$ ensures an interior solution. Then

$$
\begin{aligned}
x^P - x^C &= \frac{A}{\theta\Phi}\left\{(2 - (1 + \beta)\gamma)\theta - 2(2 - \beta\gamma)\Phi\right\} \\
&= \frac{Ab\delta}{\theta\Phi}\left\{(2 - \gamma)(2 + \gamma)^2(2 - (1 + \beta)\gamma) - 8(1 - \gamma)(1 + \gamma)^2(2 - \beta\gamma)\right\} \\
&= \frac{Ab\delta}{\theta\Phi}\left\{4(1 - \beta)\gamma^2 + 2\gamma^2(2 + \gamma^2 - \gamma - 2\beta\gamma) + (2 - \gamma)(1 - \beta)\gamma^3\right\}
\end{aligned}
$$

Because the expression within brackets is positive for all β and for all $\gamma > 0$, this proves the proposition. *Q.E.D.*

Another Perspective on Proposition 2

For two values of γ, 0.4 and 0.6, we plot percentage differences in both output and profit (*e.g.* $\frac{q^P - q^C}{q^C} \times 100\%$ for output) against the spillover parameter β. In the top panel of Figure 3 and the first column of Table 1, for $\gamma = 0.4$, we see that both output and profit are lower under the cartel if β is less than about 0.1, which corresponds to a slice of region B in Figure 2. But cartel and competition are not much different in this interval, as cartel output is between 5% and 10% less than competitive output, and cartel profit is no more than 5% less than competitive profit. In the bottom panel of Figure 3 and the second column of Table 1, for $\gamma = 0.6$, both output and profit are higher under the cartel if β is between 0.3 and 0.7, which is a slice of region A. In this interval, output is not significantly higher for a cartel, with cartel output exceeding competitive output by around 5%; however, cartel profit exceeds competitive profit by as much as 35% in this interval.

Table 1. Percent Differences Between Regimes P (Production Cartel) and C (Competition)

β	$\gamma = 0.4$		$\gamma = 0.6$	
	Output	*Profit*	*Output*	*Profit*
0.0	-7.39	-4.17	3.52	-81.40
0.1	-6.78	0.69	5.15	-49.52
0.2	-6.25	5.08	6.28	-22.51
0.3	-5.81	8.96	6.78	-0.31
0.4	-5.51	12.27	6.51	16.92
0.5	-5.36	14.96	5.40	29.00
0.6	-5.41	16.94	3.41	35.93
0.7	-5.69	18.10	0.60	37.98
0.8	-6.25	18.35	-2.95	35.78
0.9	-7.12	17.62	-7.06	30.19
1.0	-8.33	15.85	-11.54	22.25

Figure 3