Supplemental material to the paper "A Search Model Where Consumers Choose Quantity Based on Average Price" by Paolo Buccirossi

In this supplemental material I extend the result of the paper to the case where some consumers are perfectly informed or, equivalently, have no search costs. These consumers choose the firm that charges the lowest price and base their quantity decision on this price.

The model presented in section 2 of the paper is modified as follows.

There are k consumers, with k normalized to 1. They come in two types: Informed Consumers (IC) and Uninformed Consumers (UC). The parameter β denotes the proportion of IC and is exogenously given. Consumers are characterized by a function $p_k : R^n_+ \to R_+$, where p_k is the perceived price for consumer k given the firms' strategy profile. For IC we have:

(1) $$p_{IC} = \min\{p\},$$

whereas for UC:

(2) $$p_{UC} \equiv p^e = \sum_{i=1}^{n} \alpha_i p_i ,$$

where α_i is the share of UC served by firm i. I will refer to p^e also as "the expected price". Note, however, that this is not the average market price because it does not consider the price paid by IC.

All consumers possess an identical concave and decreasing demand function, $q_k = q(p_k)$, with $q(p_k) = 0$ for $p_k \geq \bar{p}$ .Therefore, $pq(p)$ is strictly concave, and I denote with p^m its unique maximizer. Market demand is $q(p, \beta, \alpha) = \beta q(p_{IC}) + (1- \beta)q(p^e)$.

Firm i's demand depends on firms strategies and on consumers behavior. Given a strategy profile, p, let $m(p)$ be the number of firms that charges the lowest price. IC purchase from these firms. If more than one firm has the lowest price, demand is divided equally among them.[1] Hence, firm i gets the following market share of IC:

$$\gamma_i = \begin{cases} 1/m & \text{if } p_i = \min\{p\} \\ 0 & \text{otherwise} \end{cases}.$$

[1] Equality of the market shares of IC among the firms that charge the lowest price is assumed for the sake of exposition. The adoption of any other sharing rules does not affect the results of the model.

UC respond to a modification of strategies adopted by firms with a modification of quantity demanded, but without switching from one firm to another.

Therefore, every firm faces the following demand function:

(3) $q_i(p) = \gamma_i(p)\beta q(p_i) + \alpha_i(1 - \beta)q(p^e)$.

Firm i's profits are:

(4) $\pi_i = p_i[\gamma_i(p)\beta q(p_i) + \alpha_i(1 - \beta)q(p^e)]$.

Since with $\beta > 0$ payoff functions are not continuous, as γ_i is not continuous in p_i, an equilibrium in pure strategies may not exist. To show that a pure strategy equilibrium exists in some cases, I first prove the following Lemma.

Lemma 1. If $0 < \beta < 1$ and a pure strategy equilibrium, p^*, exists, it must be $m(p^*) = 1$.

Proof. Suppose p_i^* is the lowest price in the equilibrium profile and $m(p^*) > 1$. We can have two cases: 1) $p_i^* > 0$; 2) $p_i^* = 0$. In case 1, slightly cutting its price, firm i's profits make a jump up since it gets all IC, whereas before it had only a share, $1/m < 1$, of IC. In case 2, there is always a positive ε, such that $p^e < \overline{p}$, thus charging ε, it still gets a strictly positive demand from UC and makes positive profits whereas at $p_i^* = 0$ its profits were zero.

The following Lemma proves that an equilibrium in pure strategies exists in some cases.

Lemma 2. If $0 < \beta < 1$ and there exist a α_i such that $\alpha_i > \alpha_j$ for any $j \neq i$, a pure strategy equilibrium exists provided that β is sufficiently small.

Proof. Denote with i the firm with the highest market share and therefore with the lowest price. From the previous Lemma we know that i is the only firm charging this price and that it serves all the IC. Thus, it gets the following profits

(5) $p_i[\beta q(p_i) + \alpha_i(1 - \beta)q(p^e)]$.

Which are maximized for

(6) $$p_i = \frac{\alpha_i(1-\beta)q(p^e)+\beta a(p^e)}{\alpha_i^2(1-\beta)\dfrac{\partial q(p^e)}{\partial p^e}+\beta\dfrac{\partial q(p_i)}{\partial p_i}},$$

which is the implicit price function for firm *i*. Firms $j \neq i$ sell only to UC. As they do not have the lowest price, their payoff functions are:

(7) $$\pi_j = p_j\alpha_j(1 - \beta)q(p^e).$$

Their implicit price functions, obtained from the FOCs for the maximization of their profits, are defined as follows:

(8) $$p_j = -\frac{q}{\alpha_j}\frac{dp}{dq}.$$

Let us indicate with $p^* = (p_i^*, p_{-1}^*)$ a point in R_+^n that simultaneously satisfies Equations (7) and (8). The vector p^* is a candidate equilibrium.

Now I need to prove that all the implicit price functions are reaction functions. Let us verify this for firm *i*. If only firm *i* deviates by raising its price, it reduces its profits in any case. Indeed, if its price remains below the lowest price charged by the rival firms its demand does not change and the maximizing price remains p_i^*. As soon as the price becomes equal to the price of a competitor, firm *i* looses some IC and therefore its profits make a jump down. Finally, if firm *i* decides to charge a price above that of at least one competitor it will chose a price that maximizes the profit function with only IC. However, since *i* has the largest market share its optimal price will be the lowest price in the market. This contradicts the hypothesis that firm *i* charges a price above the price charged by at least one of its competitors. Hence firms *i* has no reason to deviates unilaterally from p_i^*.

Let us consider firms *j*. None of the high price firms must find profitable to cut their price in order to capture IC. Suppose that, while all the other firms price according to p^*, firm *j* charges p_i^* and gets all IC. If, by doing so, its profits were strictly greater than the profits it would obtain charging p_j^*, then there would be an $\varepsilon > 0$ such that, setting its price equal to p_i^* - ε, it would actually increase its profits and therefore p_j^* would not be its best response to p_{-j}^*. We have to find under what conditions such a possibility does not exist for every *j*. The following inequalities must be satisfied:

(9) $$p_j^* \alpha_j (1-\beta) q(p^{e*}) \geq p_i^* \left[\alpha_j (1-\beta) q(p^{ej}) + \beta q(p_i^*)\right]$$

where p^{ej} is the expected price that we get if all firms charge the price defined by the candidate equilibrium, p^*, except for firm *j* that matches the lowest price in the market, p_i^*. We know that $p^{ej} \geq p_i^*$, and therefore that $q(p^{ej}) \leq q(p_i^*)$. Hence,

(10) $$p_i^* q(p_i^*)\left[\alpha_j (1-\beta) + \beta\right] \geq p_i^* \left[\alpha_j (1-\beta) q(p^{ej}) + \beta q(p_i^*)\right].$$

It follows that if

(11) $$p_j^* \alpha_j (1-\beta) q(p^{e*}) \geq p_i^* q(p_i^*)\left[\alpha_j (1-\beta) + \beta\right],$$

condition (9) is surely satisfied. Condition (11) can be written as:

(12) $$\frac{p_j^* q(p^{e*})}{p_i^* q(p_i^*)} \geq \frac{\alpha_j (1-\beta) + \beta}{\alpha_j (1-\beta)}.$$

Now we have that, if $\alpha_i > \alpha_j$,

(13) $$\lim_{\beta \to 0} \frac{p_j^* q(p^{e*})}{p_i^* q(p_i^*)} > 1,$$

whereas

(14) $$\lim_{\beta \to 0} \frac{\alpha_j (1-\beta) + \beta}{\alpha_j (1-\beta)} = 1.$$

Therefore there exists an interval $[0, \beta^*]$, where β^* is only implicitly defined by (3), such that for any β in this interval condition (4) is satisfied for any *j* and the strategy profile p^* constitutes a Nash equilibrium.

Finally I can characterize the expected equilibrium price.

Proposition If $0 < \beta < 1$ and a pure strategy equilibrium exists, then the expected equilibrium price is above the monopoly level.

Proof. Denote with *i* the firm with the lowest price. Given equations (2), we can write:

(15) $$\sum_{f=1}^{n} \alpha_f p_f^* = -(n-1) q(p^{e*}) \frac{\partial p^e}{\partial q(p^e)} + \alpha_i p_i^* ,$$

that after some manipulation, becomes:

(16) $$\varepsilon(p^{e*}) = (n-1) \frac{p^{e*}}{p^{e*} - \alpha_i p_i^*} ,$$

where $\varepsilon(p^{e*})$ is the absolute value of the elasticity of demand computed at the expected equilibrium price. Since

(17) $$\frac{p^{e*}}{p^{e*} - \alpha_i p_i^*} > 1 ,$$

we can say that, for any $n \geq 2$, in equilibrium $\varepsilon(p^{e*}) > 1$, and therefore that $p^{e*} > p^m$.