Supplemental Material for Yongmin Chen, "Strategic Bidding by Potential Competitors: Will Monopoly Persist," *The Journal of Industrial Economics*, 48 (2), June 2000, pp.161-175.

0.1 Appendix

Proof of Proposition 1. If $K > \max\{\Pi_N(XY, X), \Pi_N(X, XY) - \Pi_N(X, Y)\}$, in equilibrium N will not enter X whether M or N wins the bidding for Y. By Lemma 2, $\Pi_M(XY, X) = \Pi_N(X, XY)$. Therefore if $K > \max\{\Pi_N(XY, X), \Pi_M(XY, X) - \Pi_N(X, Y)\}$, $v_M = \Pi_M(XY, 0) - \Pi_M(X, Y)$ and $v_N = \Pi_N(X, Y)$. Since by Lemma 1,

$$\Pi_M(XY, 0) > \Pi_M(X, Y) + \Pi_N(X, Y),$$

M wins the bidding for Y and the winning bid is $\Pi_N(X, Y)$. ■

Proof of Proposition 2. When $\Pi_N(XY, X) > K > \Pi_N(X, XY) - \Pi_N(X, Y)$, in equilibrium N will stay out of X if winning Y, but N will enter X if M wins Y. In this case, $v_M = \Pi_M(XY, X) - \Pi_M(X, Y)$ and $v_N = \Pi_N(X, Y) - (\Pi_N(XY, X) - K)$. By Lemma 2, $\Pi_M(XY, X) = \Pi_N(X, XY)$. Thus when $\Pi_N(XY, X) > K > \Pi_M(XY, X) - \Pi_N(X, Y)$, N wins Y if

$$\Pi_N(X, Y) - \Pi_N(XY, X) + K > \Pi_M(XY, X) - \Pi_M(X, Y),$$

which holds if

$$\Pi_N(X, Y) - \Pi_N(XY, X) + \Pi_M(XY, X) - \Pi_N(X, Y) \geq \Pi_M(XY, X) - \Pi_M(X, Y),$$

or $\Pi_M(X, Y) \geq \Pi_N(XY, X)$.

Next, in equilibrium, when $\Pi_N(X, XY) - \Pi_N(X, Y) > K > \Pi_N(XY, X)$, N will enter X if winning Y, but N will not enter X if M wins the bidding. In this case, $v_M = \Pi_M(XY, 0) - \Pi_M(X, XY)$ and $v_N = \Pi_N(X, XY) - K$. Therefore, when $\Pi_M(XY, X) - \Pi_N(X, Y) > K > \Pi_N(XY, X)$, M wins Y and N stays out of X if

$$\Pi_M(XY, 0) - \Pi_M(X, XY) > \Pi_N(X, XY) - K,$$

which holds by combining the following: $\Pi_M(XY, 0) > \Pi_M(XY, X) + \Pi_N(XY, X) - K$ since $\Pi_M(XY, 0) > \Pi_M(XY, X)$ from Lemma 1; $\Pi_M(XY, X) = \Pi_N(X, XY)$ and $\Pi_N(XY, X) = \Pi_M(X, XY)$ from Lemma 2. ■

Proof of Corollary 1. Simple calculations reveal the following:

$$\Pi_M(X,Y) = \left(\frac{1}{2-\beta}\right)^2, \qquad \Pi_N(X,Y) = \left(\frac{1}{2-\beta}\right)^2;$$

$$\Pi_M(XY,X) = \frac{1}{36}\frac{(5\beta+13)}{(1-\beta)}, \qquad \Pi_N(XY,X) = \frac{1}{9}.$$

Therefore,

$$\Pi_M(X,Y) - \Pi_N(XY,X) = \left(\frac{1}{2-\beta}\right)^2 - \frac{1}{9},$$

which increases in β for $\beta \in (-1,1)$. Since $\Pi_M(X,Y) - \Pi_N(XY,X) = 0$ when $\beta = -1$, $\Pi_M(X,Y) \geq \Pi_N(XY,X)$ always holds in this example. Next,

$$\Pi_M(XY,X) - \Pi_N(X,Y) = \frac{1}{36}\frac{(\beta+1)(5\beta^2 - 12\beta + 16)}{(1-\beta)(2-\beta)^2},$$

$$\Pi_M(XY,X) - (\Pi_N(XY,X) + \Pi_N(X,Y)) = \frac{1}{4}\beta\frac{4-3\beta+\beta^2}{(1-\beta)(2-\beta)^2}.$$

Since, for $\beta \in (-1,1)$,

$$\frac{4-3\beta+\beta^2}{(1-\beta)(2-\beta)^2} > 0,$$

we have $\Pi_M(XY,X) - \Pi_N(X,Y) < \Pi_N(XY,X)$ if and only if $\beta < 0$, and $\Pi_M(XY,X) - \Pi_N(X,Y) < \Pi_N(XY,X)$ if and only if $\beta > 0$. The conclusion then follows from Proposition 2. ■

Proof of Theorem 1. Under *A1* and by Proposition 2, we need only to prove the following two claims. *Claim 1*: $\Pi_N(XY,X) > \Pi_M(XY,X) - \Pi_N(X,Y)$ and $\Pi_M(X,Y) \geq \Pi_N(XY,X)$ if X and Y are strategic substitutes, and *Claim 2*: $\Pi_N(XY,X) < \Pi_M(XY,X) - \Pi_N(X,Y)$ if X and Y are strategic complements.

Proof of Claim 1: First,

$$\Pi_N(XY,X) = x_N^2 f(x_M^2 + x_N^2, y_M^2),$$

$$\Pi_M(XY,X) = x_M^2 f(x_M^2 + x_N^2, y_M^2) + y_M^2[g(y_M^2, x_M^2 + x_N^2) - c],$$

$$\Pi_M(X,Y) = x_M^1 f(x_M^1, y_N^1), \qquad \Pi_N(X,Y) = y_N^1[g(y_N^1, x_M^1) - c].$$

In equilibrium, the following first-order conditions must be satisfied:

$$f(x_M^2 + x_N^2, y_M^2) + x_N^2 f_1(x_M^2 + x_N^2, y_M^2) = 0, \quad (3)$$

$$f(x_M^2 + x_N^2, y_M^2) + x_M^2 f_1(x_M^2 + x_N^2, y_M^2) + y_M^2 g_2(y_M^2, x_M^2 + x_N^2) = 0, \quad (4)$$

$$x_M^2 f_2(x_M^2 + x_N^2, y_M^2) + g(y_M^2, x_M^2 + x_N^2) - c + y_M^2 g_1(y_M^2, x_M^2 + x_N^2) = 0, \quad (5)$$

$$f(x_M^1, y_N^1) + x_M^1 f_1(x_M^1, y_N^1) = 0, \quad (6)$$

$$g(y_N^1, x_M^1) - c + y_N^1 g_1(y_N^1, x_M^1) = 0. \quad (7)$$

Since $g_2 < 0$ when X and Y are strategic substitutes, we have $x_M^2 < x_N^2$. Hence

$$x_M^2 f(x_M^2 + x_N^2, y_M^2) < \Pi_N(XY, X).$$

It then follows that $\Pi_N(XY,X) + \Pi_N(X,Y) > \Pi_M(XY,X)$ if

$$y_N^1[g(y_N^1, x_M^1) - c] \geq y_M^2[g(y_M^2, x_M^2 + x_N^2) - c].$$

Now if $x_M^1 \leq x_M^2 + x_N^2$, then

$$y_N^1[g(y_N^1, x_M^1) - c] \geq y_M^2[g(y_M^2, x_M^1) - c] \geq y_M^2[g(y_M^2, x_M^2 + x_N^2) - c].$$

Therefore our proof of the first part of *Claim 1* will be complete if we can show that $x_M^1 \leq x_M^2 + x_N^2$. Suppose to the contrary, $x_M^1 > x_M^2 + x_N^2$. Then, since

$$\frac{\partial \pi_x(x_M^2 + x_N^2, y_M^2)}{\partial x} = f(x_M^2 + x_N^2, y_M^2) + (x_M^2 + x_N^2) f_1(x_M^2 + x_N^2, y_M^2) < 0$$

from equations (3) and (6), we have, from the property of strategic substitutes, $y_M^2 > y_N^1$.

Next, by the intermediate-value theorem for functions of multiple variables,

$$\frac{\partial \pi_x(x_M^2 + x_N^2, y_M^2)}{\partial x} - \frac{\partial \pi_x(x_M^1, y_N^1)}{\partial x} = (x_M^2 + x_N^2 - x_M^1)\frac{\partial^2 \pi_x(x', y')}{\partial x^2} + (y_M^2 - y_N^1)\frac{\partial^2 \pi_x(x', y')}{\partial x \partial y}$$

for some $x_M^2 + x_N^2 < x' < x_M^1$ and $y_N^1 < y' < y_M^2$. But since $\frac{\partial \pi_x(x_M^2 + x_N^2, y_M^2)}{\partial x} < 0$ and $\frac{\partial \pi_x(x_M^1, y_N^1)}{\partial x} = 0$, we have

$$-(x_M^1 - (x_M^2 + x_N^2))\frac{\partial^2 \pi_x(x', y')}{\partial x^2} < -(y_M^2 - y_N^1)\frac{\partial^2 \pi_x(x', y')}{\partial x \partial y}.$$

Since, by assumption *A2*,

$$-\frac{\partial^2 \pi_x(x', y')}{\partial x^2} \geq -\frac{\partial^2 \pi_x(x', y')}{\partial x \partial y},$$

we have

$$(x_M^1 - (x_M^2 + x_N^2)) < (y_M^2 - y_N^1).$$

On the other hand, again by the intermediate-value theorem,

$$\frac{\partial \pi_y(y_M^2, x_M^2 + x_N^2)}{\partial y} - \frac{\partial \pi_y(y_N^1, x_M^1)}{\partial y} = (y_M^2 - y_N^1)\frac{\partial^2 \pi_y(y'', x'')}{\partial y^2} + (x_M^2 + x_N^2 - x_M^1)\frac{\partial^2 \pi_y(y'', x'')}{\partial y \partial x}$$

for some $x_M^2 + x_N^2 < x'' < x_M^1$ and $y_N^1 < y'' < y_M^2$. But since

$$\frac{\partial \pi_y(y_M^2, x_M^2 + x_N^2)}{\partial y} = g(y_M^2, x_M^2 + x_N^2) - c + y_M^2 g_1(y_M^2, x_M^2 + x_N^2) > 0$$

from equation (5), and $\frac{\partial \pi_y(y_N^1, x_M^1)}{\partial y} = 0$, we have

$$-(y_M^2 - y_N^1)\frac{\partial^2 \pi_y(y'', x'')}{\partial y^2} < -(x_M^1 - (x_M^2 + x_N^2))\frac{\partial^2 \pi_y(y'', x'')}{\partial y \partial x}.$$

Since

$$-\frac{\partial^2 \pi_y(y'', x'')}{\partial y^2} \geq -\frac{\partial^2 \pi_y(y'', x'')}{\partial y \partial x}$$

from *A2*, we have

$$(y_M^2 - y_N^1) < (x_M^1 - (x_M^2 + x_N^2)).$$

This is a contradiction. Therefore $x_M^1 \leq x_M^2 + x_N^2$.

Next, for the second part of *Claim 1*,

$$\Pi_M(X, Y) = x_M^1 f(x_M^1, y_N^1) \geq x_N^2 f(x_N^2, y_N^1).$$

Since

$$x_N^2 f(x_N^2, y_N^1) - x_N^2 f(x_M^2 + x_N^2, y_M^2) = x_N^2 \left[f(x_N^2, y_N^1) - f(x_M^2 + x_N^2, y_M^2) \right],$$

it follows that $\Pi_M(X, Y) \geq \Pi_N(XY, X)$ if $f(x_N^2, y_N^1) - f(x_M^2 + x_N^2, y_M^2) \geq 0$. Since $x_M^1 \leq x_M^2 + x_N^2$, we have $y_M^2 < y_N^1$ from equations (5) and (7). By the intermediate-value theorem, there exists some $(\widetilde{x}, \widetilde{y})$ with $x_N^2 < \widetilde{x} < x_M^2 + x_N^2$ and $y_M^2 < \widetilde{y} < y_N^1$ such that

$$f(x_N^2, y_N^1) - f(x_M^2 + x_N^2, y_M^2) = -x_M^2 f_1(\widetilde{x}, \widetilde{y}) + (y_N^1 - y_M^2) f_2(\widetilde{x}, \widetilde{y}),$$

which is non-negative by assumption *A3*. This completes the proof of *Claim 1.*

Proof of Claim 2:
$\Pi_M(XY, X) - (\Pi_N(XY, X) + \Pi_N(X, Y))$
$= x_M^2 f(x_M^2 + x_N^2, y_M^2) - x_N^2 f(x_M^2 + x_N^2, y_M^2) + y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c].$
Since from equations (3) and (4),

$$x_M^2 f(x_M^2 + x_N^2, y_M^2) - x_N^2 f(x_M^2 + x_N^2, y_M^2) = x_N^2 y_M^2 g_2(y_M^2, x_M^2 + x_N^2),$$

$$\begin{aligned} & \Pi_M(XY, X) - (\Pi_N(XY, X) + \Pi_N(X, Y)) \\ = \; & x_N^2 y_M^2 g_2(y_M^2, x_M^2 + x_N^2) + y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c]. \end{aligned}$$

If $y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c] \geq 0$, our proof is complete. Now suppose

$$y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c] < 0.$$

If $y_M^2 \leq y_N^1$, then from (5) and (7), we would have $x_M^2 + x_N^2 < x_M^1$. But then M could increase profit by choosing $x_M^2 = x_M^1 - x_N^2$ and $y_M^2 = y_N^1$. Thus $y_M^2 > y_N^1$, which implies $x_M^2 + x_N^2 > x_M^1$ from comparing equations (3) and (6), using the property of strategic complements. Now

$$\begin{aligned} & y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c] \\ > \; & y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^2 + x_N^2) - c] \\ = \; & (y_M^2 - y_N^1)[g(\widehat{y}, x_M^2 + x_N^2) - c + \widehat{y} g_1(\widehat{y}, x_M^2 + x_N^2)] \end{aligned} \tag{8}$$

for some $\widehat{y} \in (y_N^1, y_M^2)$, where the inequality is due to $g_2 > 0$ and the equality is due to the intermediate-value theorem. we thus must have

$$g(\widehat{y}, x_M^2 + x_N^2) - c + \widehat{y} g_1(\widehat{y}, x_M^2 + x_N^2) < 0.$$

Since $\widehat{y} < y_M^2$, it then follows that

$$g(y_M^2, x_M^2 + x_N^2) - c + y_M^2 g_1(y_M^2, x_M^2 + x_N^2) < g(\widehat{y}, x_M^2 + x_N^2) - c + \widehat{y} g_1(\widehat{y}, x_M^2 + x_N^2). \tag{9}$$

Therefore,

$$
\begin{aligned}
& \Pi_M(XY,X) - (\Pi_N(XY,X) + \Pi_N(X,Y)) \\
= \;& x_N^2 y_M^2 g_2(y_M^2, x_M^2 + x_N^2) + y_M^2[g(y_M^2, x_M^2 + x_N^2) - c] - y_N^1[g(y_N^1, x_M^1) - c] \\
> \;& x_N^1 y_M^2 g_2(y_M^2, x_M^2 + x_N^2) + (y_M^2 - y_N^1)[g(y_M^2, x_M^2 + x_N^2) - c + y_M^2 g_1(y_M^2, x_M^2 + x_N^2)] \\
= \;& x_N^2 y_M^2 g_2(y_M^2, x_M^2 + x_N^2) - (y_M^2 - y_N^1) x_M^2 f_2(x_M^2 + x_N^2, y_M^2) \\
\geq \;& 0,
\end{aligned}
$$

where the first inequality is due to relations (8) and (9), the second equality is due to equation (5), and the last inequality is due to assumption *A3*.

■