Supplemental Material for J. D. Dana, Jr. and K. E. Spier, "Revenue Sharing and Vertical Control in the Video Rental Industry," *The Journal of Industrial Economics* ********* .

Proof of Equation (7):

Let the solution to this program be $\{P', K'\}$ and suppose it is *not* the competitive equilibrium. Then there exists another *market price and inventory*, $\{\hat{P}, \hat{K}\}$, at which consumers would obtain strictly greater surplus, $(V-\hat{P})S(\hat{K}) > (V-\tilde{P})S(\tilde{K})$, and firms would earn strictly greater profits than at $\{\tilde{P}, \tilde{K}\}$. Starting at the competitive equilibrium, $\{\tilde{P}, \tilde{K}\}$, suppose that an "entrant" offers $\{\hat{P}, k\}$, that is, an *additional k* units of inventory at price $\hat{P}$. A sufficient condition for this to be a profitable strategy is that the entrant's allocation of customers, θ, is greater than $k/\hat{K}$. To prove that this is indeed true, all we need to show is that if the entrant's allocation were exactly $k/\hat{K}$, then consumers would get higher surplus from the entrant than from the rest of the market. Under the proposed allocation, $\theta = k/\hat{K}$, a consumer allocated to the entrant gets expected surplus $(V-\hat{P})S(\hat{K})/E(x)$ (that is, the same surplus that would be obtained if the whole market offered $\{\hat{P}, \hat{K}\}$. A consumer allocated to $\{\tilde{P}, \tilde{K}\}$, on the other hand, receives expected surplus

$$(V-\tilde{P})\left\{\frac{\int_0^{\tilde{K}/(1-\theta)}(1-\theta)xf(x)dx + \int_{\tilde{K}/(1-\theta)}^{\infty}\tilde{K}f(x)dx}{(1-\theta)E(x)}\right\},$$

since now $(1-\theta)x$ consumers instead of x consumers are chasing the $\tilde{K}$ units of capacity. This expression approaches $(V-\tilde{P})S(\tilde{K})/E(x)$ as k approaches zero (because $\theta = k/\hat{K}$ approaches zero). Finally, using the assumption that $(V-\hat{P})S(\hat{K}) > (V-\tilde{P})S(\tilde{K})$ establishes that the consumers are strictly better off with $\{\hat{P}, k\}$ than with $\{\tilde{P}, \tilde{K}\}$ under the proposed allocation. Therefore $\theta > k/\hat{K}$ and we are done: $\{\tilde{P}, \tilde{K}\}$ is not a competitive equilibrium.