Supplemental Materials for Vincenzo Denicolò, "Compatibility and Bundling with Generalist and Specialist Firms," *The Journal of Industrial Economics* 48 (2), June 2000, pp. 177-187

1 I. Proofs of Lemmas

Proof of Lemma 1. Assuming $p_2^X + p_3^Y \leq p_1$, the consumer located at $(0,1)$ will always purchase the system offered by the specialist firms since $p_2^X + p_3^Y + 1 + a^2 - 2a \leq p_1 + 1 + a^2$. Two sub-cases are then possible, depending on which system is purchased by the consumer located at $(1,0)$. When $p_1 - p_2^X - p_3^Y \leq 2a$, he will purchase the system offered by firm 1, and total demand for this system is $q_1 = \left(2 + p_2^X + p_3^Y - p_1 - 2a\right)^2/(8 - 16a)$. The three firms' reaction functions have only one intersection point which is $p_1 = (1 - a + \sqrt{a^2 - 42a + 21})/5$ and $p_2^X = p_3^Y = (-7 + 7a + 3\sqrt{a^2 - 42a + 21})/10$. However, simple algebra confirms that $p_1 - p_2^X - p_3^Y = (8 - 8a - 2\sqrt{a^2 - 42a + 21})/5 > 2a$, thus there is no pure-strategy equilibrium in the region $p_1 - p_2^X - p_3^Y \leq 2a$. When instead $p_1 - p_2^X - p_3^Y > 2a$, the consumer located at $(0,1)$ will purchase the system offered by the specialist firms, and the demand for the system produced by firm 1 is $q_1 = \left(1 + p_2^X + p_3^Y - p_1\right)/2$. The three firms' reaction functions have only one intersection point which is $p_1 = \frac{5}{4}$ and $p_2^X = p_3^Y = \frac{3}{4}$, violating the condition $p_1 \geq p_2^X + p_3^Y$. Thus there is no equilibrium with $p_1 \geq p_2^X + p_3^Y$. ■

Proof of Lemma 2. When the demand function is given by (4), the three firms' reaction functions are given by $p_1 = \left(1 + p_2^X + p_3^Y\right)/2$ for firm 1, $p_2^X = \left(1 + p_1 - p_3^Y\right)/2$ for firm 2, and $p_3^Y = \left(1 + p_1 - p_2^X\right)/2$ for firm 3. These reaction curves have only one intessection point which is $p_1 = \frac{5}{4}$ and $p_2^X = p_3^Y = \frac{3}{4}$. This candidate equilibrium, however, is consistent with the assumption that $p_2^X + p_3^Y - p_1 \leq 2a$ only if $a \geq \frac{1}{8}$.

When the demand function is given by (3), the three firms' reaction curves have only one intersection point which is $p_1 = (-6 + 6a + 4\sqrt{a^2 - 22a + 11})/5$ and $p_2^X = p_3^Y = (1 - a + \sqrt{a^2 - 22a + 11})/5$. This candidate equilibrium, however, is consistent with the assumption that $p_2^X + p_3^Y - p_1 \geq 2a$ only if $a \leq \frac{1}{8}$. ■

2 II. Analysis of the shifts in reaction functions

Compatibility. Under compatibility the markets for the two components are uncoupled. In the market for component Y, reaction curves are

$$p_1^Y = (1 + p_3^Y)/2$$

and

$$p_3^Y = (1 + p_1^Y)/2.$$

In the market for component X, reaction curves are

$$p_1^X = (1 - 2a + p_2^X)/2$$

and

$$p_2^X = (1 - 2a + p_1^X)/2.$$

In each market, a symmetric Hotelling-type equilibrium is reached: $p_1^X = p_2^X = 1 - 2a$ and $p_1^Y = p_3^Y = 1$.

Incompatibility. When $p_2^X + p_3^Y - p_1 > 2a$, which holds in equilibrium iff $a < 1/8$, we have:

$$q_1 = \frac{1 + p_2^X + p_3^Y - p_1}{2} - \frac{\left(p_2^X + p_3^Y - p_1 - 2a\right)^2}{8 - 16a},$$

whereas if $p_2^X + p_3^Y - p_1 < 2a$, which holds in equilibrium iff $a \geq 1/8$, the demand for the system produced by firm 1 is:

$$q_1 = \frac{1 + p_2^X + p_3^Y - p_1}{2}.$$

I focus on two case, $a = 0$ and $a \geq 1/8$ (the same qualitative results hold for $0 < a < 1/8$). The reaction functions are as follows. When $a = 0$, firm 1's reaction curve is, in implicit form,

$$4 - 8p_1 - 3p_1^2 + 4p_1(p_2^X + p_3^Y) + 4(p_2^X + p_3^Y) - (p_2^X + p_3^Y)^2 = 0.$$

Firms 2 and 3's reaction curves are:

$$p_2^X = \frac{2 + p_1 - p_3^Y}{3}$$

and

$$p_3^Y = \frac{2 + p_1 - p_2^X}{3}$$

respectively.

When $a \geq 1/8$, one gets:

$$p_1 = \frac{1 + p_2^X + p_3^Y}{2},$$

for firm 1, and

$$p_2^X = \frac{1 + p_1 - p_3^Y}{2},$$

$$p_3^Y = \frac{1 + p_1 - p_2^X}{2},$$

for firms 2 and 3, respectively.

Comparison. Comparing reaction curves under compatibility and incompatibility is not a trivial task, since firms react to different variables depending on the mode of competition. For instance, firm 2 reacts to p_1^X under compatibility whereas it reacts to p_1 and p_3^Y under incompatibility. One way to perform the comparison is to aggregate firms' reaction curves into the *system reaction curves* that represent the price of each system as a function of the price of the competing system. For instance, summing the reaction functions under compatibility one obtains:

$$p_1^X + p_1^Y = \frac{2(1-a) + p_2^X + p_3^Y}{2},$$

$$p_2^X + p_3^Y = \frac{2(1-a) + p_1^X + p_1^Y}{2}.$$

Under incompatibility, firm 1's reaction curve is effectively a system reaction function, while firms 2 and 3's reaction curves can be summed up to get:

$$p_2^X + p_3^Y = \frac{2 + p_1}{2}$$

when $a = 0$, and

$$p_2^X + p_3^Y = \frac{2(1 + p_1)}{3}$$

when $a \geq 1/8$. These can be interpreted as firms 2 and 3's system reaction curve. It is clear that the specialist firms' system reaction curve does not change when $a = 0$, whereas these firms are softer in the aggregate when $a \geq 1/8$. Firm 1 is always softer under incompatibility, except in the case $a = 1/2$ when its system reaction curve is the same independently of the compatibility choice.

Another way to compare the reaction curves, which helps explain why the specialist firms are softer in the aggregate under incompatibility, is the following. Under incompatibility, one can substitute condition $p_1^Y = p_3^Y$ into firm 2's reaction curve, obtaining (assuming $a > 1/8$ to fix ideas):

$$p_2^X = \frac{1 + p_1^X}{2}.$$

This *component-wise reaction curve* may be interpreted as firm 2's response to firm 1's pricing of component X, *on the presumption that firm 1's pricing of component Y will exactly match p_3^Y*. The component-wise reaction curve can be directly compared to firm 2's reaction curve under compatibility, namely $p_2^X = (1 - 2a + p_1^X)/2$. Similarly, substituting condition $p_1^X = p_2^X$ into firm 3's reaction function we obtain firm 3's component-wise reaction curve, $p_3^Y = (1 + p_1^Y)/2$. It is clear that firm 2 is softer under incompatibility, while firm 3's component-wise reaction curve coincides with firm 3's reaction curve under compatibility. Note also that under incompatibility the component-wise reaction curve of the two firms are identical, and coincide with the more differentiated firm's reaction curve under compatibility. This suggests that the degree of differentiation of the systems equals the maximum differentiation of the two components.

Statements contained in Table 1 and the surrounding text can be easily checked using the above system or component-wise reaction curves.

3 III. Linear transportation costs

With linear transportation costs, there is no pure-strategy price equilibrium when $\frac{1}{4} < a < \frac{1}{2}$. However, one can easily calculate the equilibrium for $a = 0$ and $a = \frac{1}{2}$. Like in Matutes and Regibeau [1988], I assume that consumers can only travel horizontally or vertically, so that a consumer located at (x, y) will incur in transportation costs $t(x + 1 - y)$ to reach the system located at $(0, 1)$. Again, one can set $t = 1$ without any further loss of generality.

Compatibility. When $a = 0$, In the market for component Y, reaction curves are

$$p_1^Y = (1 + p_3^Y)/2$$

and

$$p_3^Y = (1 + p_1^Y)/2.$$

Analogous expressions hold for component X. At equilibrium, $p_1^X = p_2^X = p_1^Y = p_3^Y = 1$. When $a = 1/2$, equilibrium prices of component Y do not change, but one gets $p_1^X = p_2^X = 0$ because the X component is now homogeneous. Equilibrium profits are $\pi_1 = 1$ and $\pi_2 = \pi_3 = 1/2$ when $a = 0$; $\pi_1 = \pi_3 = 1/2$ and $\pi_2 = 0$ when $a = 1/2$.

Incompatibility. The indifferent consumers will be located along the line:

$$p_1 + (x - a) + y = p_2^X + p_3^Y + (1 - a - x) + (1 - y),$$

that is:

$$y = \frac{2 + p_2^X + p_3^Y - p_1}{2} - x.$$

Thus, the demand functions (and hence equilibrium prices) are independent of a. Since the analog to Lemma 1 continues to hold, we have $p_2^X + p_3^Y > p_1$ which implies that market areas will always be as in Figure 2. Demand is:

$$q_2 = q_3 = \frac{\left(2 + p_1 - p_2^X - p_3^Y\right)^2}{8},$$

and $q_1 = 1 - q_2$ since the market is covered. Reaction curves for firms 2 and 3 are $p_2^X = \left(2 + p_1 - p_3^Y\right)/3$ and $p_3^Y = \left(2 + p_1 - p_2^X\right)/3$, while firm 1's reaction curve is implicitly given by:

$$1 - \frac{\left(2 + p_1 - p_2^X - p_3^Y\right)^2}{8} - \frac{p_1\left(2 + p_1 - p_2^X - p_3^Y\right)}{4} = 0.$$

Equilibrium prices are $p_1 = (4\sqrt{11} - 6)/5$ and $p_2^X = p_3^Y = (1 + \sqrt{11})/5$. Equilibrium profits are $\pi_1 = .912$ and $\pi_2 = \pi_3 = .322$. All firms lose from incompatibility when $a = 0$, firms 1 and 2 gain, while firm 3 still loses from incompatibility when $a = 1/2$. These results parallel those obtained with quadratic transportation costs.

4 IV. Vertical product differentiation

Consider Einhorn's [1992] model of vertical product differentiation. The mass of consumers is normalized to 1. Each consumer buys one system, which is made up by two components. Components are vertically differentiated. Quality levels are exogenous. Normalizing to 0 the low quality level of each component, let q denote the high quality level of component X and Q the high quality level of component Y. Consumers' willingness to pay for quality is given by $kq + hQ$, where k and h are taste parameters which are uniformly distributed over the interval $[0, 1]$.

Einhorn distinguishes between several cases, according to whether leadership is shared or complete and the taste parameters k and h are identical or independently distributed. With generalist and specialist firms, the number of cases to analyze is still greater; for instance, with complete leadership, one must further distinguish between the case where the generalist firm leads or the specialist firms lead. To illustrate, I analyze only a few cases.

4.1 Case 1: Identical taste parameters, complete leadership, the specialist firms lead.

Compatibility. Consider the market for component X. The consumer who is indifferent between purchasing from firm 1 or firm 2 is given by condition $p_2^X - kq = p_1^X$ or

$$k = \frac{p_2^X - p_1^X}{q}.$$

Firm 1 will serve the k consumers with the lowest valuation for quality, and firm 2 will serve the $1 - k$ highest value consumers. Thus, the profit functions are $\pi_1^X = p_1^X(p_2^X - p_1^X)/q$ and $\pi_2 = p_2^X(q - p_2^X + p_1^X)/q$. Reaction curves are $p_1^X = p_2^X/2$ and $p_2^X = (q + p_1^X)/2$. Equilibrium prices are $p_1^X = q/3$ and $p_2^X = 2q/3$, implying $k = 1/3$. Equilibrium profits are $\pi_1^X = q/9$ and $\pi_2 = 4q/9$.

In the market for component Y, a similar equilibrium is reached with $\pi_1^Y = Q/9$ and $\pi_3 = 4Q/9$. Thus, profits are $\pi_1 = (q + Q)/9$, $\pi_2 = 4q/9$ and $\pi_3 = 4Q/9$. Industry profits are $5(q + Q)/9$.

Incompatibility. With $h = k$, a consumer will be indifferent between purchasing the system offered by firm 1 and that offered by firms 2 and 3 if

$p_2^X + p_3^Y - k(q+Q) = p_1$ yielding

$$k = \frac{p_2^X + p_3^Y - p_1}{q+Q}.$$

Firm 1's reaction curve is $p_1 = \left(p_2^X + p_3^Y\right)/2$, which coincides with firm 1's system reaction curve under compatibility. Firms 2 and 3's reaction curves are $p_2^X = (q+Q-p_3^Y+p_1)/2$ and $p_3^Y = (q+Q-p_2^X+p_1)/2$. These can be summed up to $p_2^X + p_3^Y = 2(q+Q+p_1)/3$, which can be compared with the system reaction curve arising under compatibility, namely $p_2^X + p_3^Y = (q+Q+p_1)/2$. Clearly, the specialist firms are softer with incompatibility.

Equilibrium prices are $p_1 = p_2^X = p_3^Y = (q+Q)/2$, whence $k = 1/2$. Equilibrium profits are $\pi_1 = \pi_2 = \pi_3 = (q+Q)/4$. Clearly, the generalist firm gains from incompatibility. This result holds independently of the asymmetry in the degree of differentiation of the two components. The specialist firms in the aggregate lose, but if one component is little differentiated, the firm producing that component may gain. Finally, industry profits are always higher under incompatibility.

4.2 Case 2: Identical taste parameters, complete leadership, the generalist firm leads.

Compatibility. Equilibrium under compatibility is the same as in case 1 except that the position of the firms is reversed. Thus, $p_1^X = 2q/3$ and $p_2^X = q/3$ in the market for component X and $p_1^Y = 2Q/3$ and $p_3^Y = Q/3$ in the market for component X. Equilibrium profits are $\pi_1 = 4(q+Q)/9$, $\pi_2 = q/9$ and $\pi_3 = Q/9$. Industry profits are $5(q+Q)/9$.

Incompatibility. With $h = k$, a consumer will be indifferent between purchasing the system offered by firm 1 and that offered by firms 2 and 3 if $p_2^X + p_3^Y = p_1 - k(q+Q)$ yielding

$$k = \frac{p_1 - p_2^X - p_3^Y}{q+Q}.$$

Firm 1's reaction curve is $p_1 = \left(p_2^X + p_3^Y + q + Q\right)/2$. Like in the previous case, this is the same as firm 1's system reaction curve under compatibility. Firms 2 and 3's reaction curves are $p_2^X = (p_1 - p_3^Y)/2$ and $p_3^Y = (p_1 - p_2^X)/2$. These can be summed up to $p_2^X + p_3^Y = 2p_1/3$, while the system reaction curve arising under compatibility is $p_2^X + p_3^Y = p_1/2$. Clearly, the specialist firms are softer with incompatibility.

Equilibrium prices are $p_1 = 3(q+Q)/4$, $p_2^X = p_3^Y = (q+Q)/4$, whence $k = 1/4$. Equilibrium profits are $\pi_1 = 9(q+Q)/16$ and $\pi_2 = \pi_3 = (q+Q)/16$. The results are the same as in case 1. The generalist firm gains from incompatibility, independently of the asymmetry in the degree of differentiation of the two components. The specialist firms in the aggregate lose, but if one component is little differentiated, the firm producing that component may gain. Finally, industry profits are higher under incompatibility.

4.3 Case 3: Independent taste parameters, complete leadership, the generalist firm leads.

Compatibility. Equilibrium under compatibility is the same as in case 2.

Incompatibility. A consumer will be indifferent between purchasing the system offered by firm 1 and that offered by firms 2 and 3 if $p_2^X + p_3^Y = p_1 - kq - hQ$. This gives:

$$k = \frac{p_1 - p_2^X - p_3^Y - hQ}{q}.$$

To proceed, we focus on the case where Q/q is sufficiently small, such that the demand for the system offered by firms 2 and 3 is given by:

$$q_2 = q_3 = \frac{p_1 - p_2^X - p_3^Y - \frac{Q}{2}}{q},$$

with $q_1 = 1 - q_2$ since the market is covered. Firm 1's reaction curve is $p_1 = \left(p_2^X + p_3^Y + q + Q/2\right)/2$. This lies below firm 1's system reaction curve under compatibility, implying that firm 1 is tougher under incompatibility. Firms 2 and 3's reaction curves are $p_2^X = (p_1 - p_3^Y + Q/2)/2$ and $p_3^Y = (p_1 - p_2^X + Q/2)/2$. These can be summed up to $p_2^X + p_3^Y = (2p_1 + Q)/3$. Since the system reaction curve arising under compatibility is $p_2^X + p_3^Y = p_1/2$, the specialist firms are softer with incompatibility.

Equilibrium prices are $p_1 = (6q+Q)/8$, $p_2^X = p_3^Y = (q - Q/2)/4$. Equilibrium profits are $\pi_1 = (6q+Q)^2/64q$ and $\pi_2 = \pi_3 = (2q-Q)^2/64q$. Industry profits are $11q/16 + Q/16 + 3Q^2/64q$. Keeping in mind that these formulas hold true when Q is sufficiently small, it follows once again that the generalist firm gains from incompatibility, the specialist firm producing the most differentiated component loses, and the specialist firm producing the least differentiated component gains. Industry profits are higher under incompatibility.

4.4 Case 4: Identical taste parameters, shared leadership, the generalist firm leads in the market for the most differentiated component.

Compatibility. Suppose that firm 1 leads in the market for component X and firm 3 leads in the market for component Y. We assume that $q > Q$, which means that component X is more differentiated than component Y. After relabelling of firms, equilibrium under compatibility is the same as in case 1. Thus, $p_1^X = 2q/3$ and $p_2^X = q/3$ in the market for component X and $p_1^Y = Q/3$ and $p_3^Y = 2Q/3$ in the market for component X. Equilibrium profits are $\pi_1 = (4q + Q)/9$, $\pi_2 = q/9$ and $\pi_3 = 4Q/9$. Industry profits are again $5(q + Q)/9$.

Incompatibility. With $h = k$, a consumer will be indifferent between purchasing the system offered by firm 1 and that offered by firms 2 and 3 if $p_2^X + p_3^Y = p_1 - k(q - Q)$. This implies:

$$k = \frac{p_1 - p_2^X - p_3^Y}{q - Q}.$$

Firm 1's reaction curve is $p_1 = \left(p_2^X + p_3^Y + q - Q\right)/2$. This lies below firm 1's system reaction curve under compatibility, i.e. firm 1 is tougher under incompatibility. Firms 2 and 3's reaction curves are $p_2^X = (p_1 - p_3^Y)/2$ and $p_3^Y = (p_1 - p_2^X)/2$. These can be summed up to $p_2^X + p_3^Y = 2p_1/3$, which lies below the system reaction curve arising under compatibility, namely $p_2^X + p_3^Y = p_1/2$. That is, the specialist firms are softer with incompatibility.

Equilibrium prices are $p_1 = 3(q - Q)/4$, $p_2^X = p_3^Y = (q - Q)/4$, whence $k = 1/4$. Equilibrium profits are $\pi_1 = 9(q-Q)/16$ and $\pi_2 = \pi_3 = (q-Q)/16$. The generalist firm gains from incompatibility only if Q is sufficiently low, i.e. $Q < \frac{17}{97}q$. Firm 2 (producing the most differentiated component) always loses from incompatibility, while firm 3 gains if $Q < \frac{9}{73}q$. Finally, industry profits are higher under incompatibility if $Q < \frac{1}{17}q$.