Supplemental Material for ANTHONY DUKES, 'THE ADVERTISING MARKET IN A PRODUCT OLIGOPOLY,' *The Journal of Industrial Economics*, VOL. 52 NO. 3 (SEPTEMBER, 2004), PP. 327-348.

In the main body of the article, we invoke the approximation $(1-\hat{\varphi}^k)^{m-1} \approx (1-\hat{\varphi}^k)^m \approx 0$. This greatly enhances the tractability of the model by permitting us to derive analytic results while preserving the economic insight. Without this approximation we are still able to characterize the same result, however, with numerical techniques.

The purpose of this appendix is to provide evidence that the approximation does not alter the main results of the basic model. We do this by first showing that the comparative statics of the equilibrium outcome with respect to the key parameters are identical in both the approximated and un-approximated models. Second we offer numerical support that welfare optimizers and associated comparative statics are only modestly affected by the approximation. We also provide intuitive explanations as to how the approximation affects the incentives of the agents.

Equilibrium outcome

We present here analytical as well as numerical evidence that the approximation does not significantly alter the qualitative aspects of the equilibrium outcome. It is readily shown that using the exact formulation of product demand in equation (3), the symmetric static equilibrium outcome yields

(S1) $$\hat{\Phi}_i^k = \frac{t_m}{mn},$$

(S2) $$\hat{p} - c = \frac{t_p}{mA(t_m/mn)}\frac{G}{H},$$

$$\hat{a} = \frac{t_p A'(t_m/mn)}{m^2 n A^2(t_m/mn)}\frac{G^2}{H},$$

$$\hat{\Pi}_i = \frac{t_p}{m^2 A(t_m/mn)}\left[1 - \left(\frac{t_m}{mn}\right)\frac{A'(t_m/mn)}{A(t_m/mn)}\right]\frac{G^2}{H} - f_p,$$

$$\hat{\Pi}^k = \frac{t_p t_m A'(t_m/mn)}{m^2 n^2 A^2(t_m/mn)}\frac{G^2}{H} - f_m,$$

where $G = 1-(1-\hat{\varphi})^m$ and $H = 1-(1-\hat{\varphi})^{m-1}$. Thus, relative to the equilibrium described in section 3.1, the equilibrium level of advertising in the competitive equilibrium remains unchanged; however, product price is scaled by the factor G/H and advertising price and profits are scaled by the factor G^2/H.

To understand this difference, first note that since the stations' optimal choice is equivalent, we need only to examine how the producer's decision is modified under the relaxed conditions $(1-\hat{\varphi})^{m-1} > 0$ and $(1-\hat{\varphi})^m > 0$. The former condition implies that fewer consumers are likely to know about competitors' products, which, on one hand, raises the producer's marginal benefit of raising its price. Consequently, producers charge a higher price, as is reflected in (S2) since $G/H > 1$. On the other hand, the condition $(1-\hat{\varphi})^m > 0$ implies that fewer consumers are participating in the product market and this tends to reduce product demand. It is ambiguous which effect dominates as illustrated by the fact that the ratio G^2/H can be greater or less than unity.

Competitive aspects among producers and stations are unaltered by the approximation. First note that both station and producer profits are subject to the same multiplicative factor, G^2/H. Thus, the presence of the approximation conditions does not alter any competitive aspects in the advertising market. Rather, invoking these conditions serves to shift some rents from consumers, in equal shares, to stations and producers when $G^2/H > 1$ and rents to consumers from stations and producers when $G^2/H < 1$.

Numerical analysis of the static equilibrium reveals only a slight discrepancy between actual and approximated profits. Table S.1 indicates a typical simulation with no more than 3% difference in profits. Given such small differences in profits in the static equilibrium, we expect free entry values of $\hat{m}$ and $\hat{n}$ to have generally no differences in light of the fact that they take integer values. Numerical calculations (not shown) support this expectation. Moreover, the comparative statics of approximated profits respect to t_p and t_m are identical to those of actual profits (see Table S.1). This implies that comparative statics of the actual free entry outcome follows as desired.

[Place Table S.I about here]

Social optimum

The only approximation used in the welfare analysis is $(1-\varphi)^m \approx 0$. We discuss this approximation with regard to the social planner's problem and then argue that the approximation only modestly affects the optimal choice (Φ^*, m^*) and that the comparative statics of this choice is unaffected by the approximation.[1]

This approximation condition implies that all consumers make a purchase in the social optimum. Relaxing this condition, $(1-\varphi)^m > 0$ yields the following welfare function:

(S3)
$$\begin{aligned} W(\Phi, m, n) &= (v_p - c)[1-(1-\Phi^\eta)^m] + (v_m - m\Phi) \\ &\quad - \frac{t_p}{4m}\left(\frac{2}{\Phi^\eta} - 1\right)[1-(1-\Phi^\eta)^m] - \frac{t_m}{4n} - mf_p - nf_m. \end{aligned}$$

Compared to the case discussed in section 4, there is one additional benefit and one additional cost associated with advertising. The benefit is that advertising not only matches consumers and products better, but also brings additional consumer to the product market. This is seen in the first term on the right-hand side of (S3). The cost is that these newly informed consumers must travel and thus incur the social cost of transportation (in addition to the reduction in programming time discussed in section 4). However, a newly informed consumer will purchase if and only if there is net positive surplus. We should expect a net increase in social welfare as a result of these additional consumers. Intuitively, therefore, we expect that the actual amount of optimal advertising to be slightly higher than in the approximated version of the model. The calculations in Table S.2 support this reasoning.

[1] Note that the approximation has no bearing on the optimal choice n^*.

Numerical simulations reveal that these additional considerations do not cause much difference in the social planner's optimal choices of Φ^*, m^*. In fact, for the most relevant variable $\Phi_S^* = m^*\Phi^*$, we see between 7% and 10% discrepancy in a typical simulation shown in Table S.2. Comparative statics with regard to t_p are consistent.

[Place Table S.II about here]

Tables: Annotated as necessary

Table S. I. Static Equilibrium Comparative Statics ($f_p = 0.10$, $f_m = 0.25$, $\eta = 0.70$)

m	n	t_p	t_m	Producer Profits $\hat{\Pi}_i$		Station Profits $\hat{\Pi}^k$	
				Actual	Approx.	Actual	Approx.
6.0	7.0	10	15	0.0712	0.0713	0.0924	0.0927
6.0	7.0	10	20	0.0404	0.0401	0.0307	0.0302
6.0	7.0	10	25	0.0199	0.0198	-0.0101	-0.0104
6.0	7.0	11	15	0.0883	0.0885	0.1267	0.1269
6.0	7.0	11	20	0.0544	0.0541	0.0588	0.0582
6.0	7.0	11	25	0.0319	0.0318	0.0139	0.0136
6.0	8.0	10	20	0.0541	0.0538	0.0196	0.0192
7.0	7.0	10	20	0.0147	0.0146	0.0177	0.0175

Table S.II. Social Optimum Comparative Statics ($f_p = 0.1$, $v_p - c = 5$, $\eta = 0.70$)

t_p	Φ^*		m^*		$\Phi_S^* = m^*\Phi^*$	
	Actual	Approx.	Actual	Approx.	Actual	Approx.
9.0	0.2629	0.2268	6.0446	6.2631	1.5889	1.4204
10.0	0.2590	0.2272	6.3595	6.5930	1.6468	1.4976
11.0	0.2553	0.2267	6.6718	6.9275	1.7032	1.5702