Supplemental Material for "Myopic Corporate Behaviour with Optimal Management Incentives," by Gerald T. Garvey, Simon Grant and Stephen P. King, *The Journal of Industrial Economics* 47 (2), June 1999, pp. 231-250

Appendix

In this appendix we show that results 1, 2, and 4 in the published article can be generalized to allow for any non-linear contract $m(V_L - V_s, V_s)$ and any continuous concave utility function $u(\cdot)$. We retain the structure of values and prices presented in section 2 in the published article but allow the independent random shocks $s$ and $z$ to be drawn from the generalized cumulative densities $F(s)$ and $G(z)$ respectively. Given a payment schedule $m(\cdot,\cdot)$ and any beliefs by the market about the future actions of the manager, we assume that there is a one-to-one relationship between $s$ and $p_s$ given a payment schedule $m(\cdot,\cdot)$ . We also assume that the relevant conditions for the first-order approach to the principal-agent problem hold through out (e.g. Laffont [1989]).

Suppose the manager cannot trade. Given rational expectations by the share market, the (risk neutral) share holders will set a contract $m(V_L - V_s, V_s) = m(z + (a - a^e), a^e + s)$ to solve

$$\max_{\langle m(\cdot,\cdot), a^e(s)\rangle} \int_z \int_s (z + s + a^e(s) - m(z, a^e(s) + s)) dF(s) dG(z) \tag{A.1}$$

subject to the incentive compatibility constraint that the manager choose the expected level of effort:

$$\forall_s \quad a^e(s) = \arg\max_a \int_z u(m(z + (a - a^e), a^e + s)) dG(z) - c(a) \tag{A.2}$$

and the individual rationality constraint

$$\int_z \int_s u(m(z, a^e(s) + s)) - c(a^e(s)) dF(s) dG(z) \geq u(\bar{Y}) \tag{A.3}$$

**Result A1** *The optimal contract for the shareholders will implement a constant action; $a(s) = \tilde{a}$ for all $s$.*

**Proof:** Suppose not and assume that the optimal contract $m(\cdot,\cdot)$ implements the (integrable) action choice function $a(s)$. Define $\bar{a} = \int_s a(s) dF(s)$. By assumption, there exists a set $\mathcal{S}$ with positive measure such that $a(s) \neq \bar{a}$ for all $s \in \mathcal{S}$. Consider a contract $\bar{m}(V_L - V_S)$ such that

$$\int_z \bar{m}(z) dG(z) = \int_z \int_s m(z, a(s) + s) dF(s) dG(z) \tag{A.4}$$

and

$$\bar{a} = \arg\max_a \int_z \bar{m}(z + a - \bar{a}) dG(z) - c(a) \tag{A.5}$$

Note that such a contract clearly exists. (A.5) can be satisfied, for example, by $\bar{m}(\cdot)$ concave with $\int_z \bar{m}'(z) dG(z) = c'(\bar{a})$. (A.4) can then be satisfied by adding or subtracting a constant payment.

Now, consider the contract $\bar{m} - \varepsilon$ where $\varepsilon$ is a small positive constant. By construction,

1. $\bar{m} - \varepsilon$ satisfies incentive compatibility to implement $\bar{a}$ for all $s$;
2. the expected long term value of the firm $V_L$ under $\bar{m} - \varepsilon$ is identical to the expected long term firm value under the original contract $m(\cdot,\cdot)$;
3. the expected payoff to the shareholders is strictly larger under $\bar{m} - \varepsilon$ than under the original contract $m(\cdot,\cdot)$; and

4. as $u$ is concave and $c$ is strictly convex then for $\varepsilon$ small enough but positive, the contract $\bar{m} - \varepsilon$ will satisfy the manager's individual rationality constraint (A.3).

As the contract $\bar{m} - \varepsilon$ solves the shareholders' program (A.1), (A.3), and (A.2) with a strictly higher payoff to the shareholders than the contract $m(\cdot,\cdot)$ then this violates the initial assumption that $m(\cdot,\cdot)$ was optimal. *Q.E.D.*

Result A1 mirrors result 1 in the published article. The corollary, that the optimal contract for the shareholders when there is no trade involves rewarding the manager only on $V_L - V_s$, follows immediately. As the action choice implemented by the shareholders is invariant in $s$, making managerial payments depend on $s$ (via payment depending on $V_s$, as revealed through the short term share price) simply adds 'noise' to the manager's contract. As the manager is risk averse and must be compensated for this 'noise', it follows from Holmström [1979] that the shareholders' optimal contract will not depend on $s$. Put simply, the optimal contract when the manager cannot trade will depend only on the gain in firm value between the short and the long term, $V_L - V_s$.

Now, allow the manager to trade a fraction $\lambda$ of her contract as before. The additional 'no trade' constraint which shareholders must consider when designing the manager's contract is

$$\forall s \quad 0 = \arg\max_{\lambda} \int_z u\left((1-\lambda)m(z, a^*(\lambda)+s) + \int_z \lambda m(z, a^*(\lambda)+s)dG(z)\right) dG(z) - c(a^*(\lambda)) \qquad \text{(A.6)}$$

The first term in the manager's utility function in (A.6) is the fraction of the contract retained by the manager after trade while the second term is the market value of the fraction of the contract sold by the manager. As in the main text, $a^*(\lambda)$ is the manager's optimal effort choice given that she has sold $\lambda$ of her contract, and by rational expectations $a^e = a^*(\lambda)$.

**Result A2** *The shareholders' optimal contract for the manager subject to the 'no trade' constraint will involve a payment scheme in which the manager's expected utility increases in the short term stock price, formally that* $\int_z \frac{\partial u}{\partial Y}\frac{\partial m}{\partial p_s} dG(z) > 0$.

**Proof:** Given the contract $m(V_L - V_s, V_s)$, from (A.6), the manager will set $\lambda$ to solve

$$\int_z \frac{\partial u}{\partial Y}\left[\begin{array}{c} -m(z, a^*+s) + (1-\lambda)\frac{\partial m}{\partial V_s}\frac{\partial a^*}{\partial \lambda} + \\ \int_z m(z, a^*+s)dG(z) + \lambda \int_z \frac{\partial m}{\partial V_s}\frac{\partial a^*}{\partial \lambda} dG(z) \end{array}\right] dG(z) - c'(a^*)\frac{\partial a^*}{\partial \lambda} = 0 \qquad \text{(A.7)}$$

To satisfy the 'no trade' constraint, $\lambda = 0$ must solve (A.7). Substituting $\lambda = 0$, (A.7) becomes

$$\int_z \frac{\partial u}{\partial Y}\left[-m(z, a^*(0)+s) + \frac{\partial m}{\partial V_s}\frac{\partial a^*(0)}{\partial \lambda} + \int_z m(z, a^*(0)+s)dG(z)\right] dG(z) = c'(a^*(0))\frac{\partial a^*(0)}{\partial \lambda} \qquad \text{(A.8)}$$

To sign the right hand side of (A.8) note that $c'(\cdot) > 0$ so we only need to sign $(\partial a^*(0)/\partial \lambda)$. Note that $a^*(\lambda)$ is given by the solution of

$$\max_a \int_z u\left((1-\lambda)m(z+a-a^e, a^e+s) + \lambda \int_z m(z, a^e+s)dG(z)\right) dG(z) - c(a)$$

The first order condition for this optimization problem is given by

$$(1-\lambda)\int_z \frac{\partial u}{\partial Y}\frac{\partial m(z+a^*-a^e, a^e+s)}{\partial(V_L-V_s)}dG(z)-c'(a^*)=0$$

with second order condition

$$\forall\lambda \quad (1-\lambda)\int_z \frac{\partial u}{\partial Y}\frac{\partial m^2(z+a^*-a^e, a^e+s)}{\partial(V_L-V_s)^2}dG(z)-c''(a^*)<0$$

and using the constraint imposed by the second order condition, it is easy to confirm that $(\partial a^*(0)/\partial\lambda)<0$. Thus the right hand side of (A.8) is negative.

The left hand side of (A.8) can be expressed as

$$\int_z \frac{\partial u}{\partial Y}\left[-m(z,a^*(0)+s)+\int_z m(z,a^*(0)+s)dG(z)\right]dG(z)+\int_z \frac{\partial u}{\partial Y}\left[\frac{\partial m}{\partial V_s}\frac{\partial a^*(0)}{\partial\lambda}\right]dG(z)$$

By strict concavity of $u(\cdot)$, the first term in this expression is greater than zero. Since $\partial a*/\partial\lambda$ is a constant, the right hand side of (A.8) can only equal the left hand side if $\int_z \frac{\partial u}{\partial Y}\frac{\partial m}{\partial V_s}dG(z)>0$. In other words, the shareholders' optimal contract subject to the manager not engaging in trade must involve a managerial payment that is increasing in the short term firm value. Given the assumption of a one-to-one relationship between $V_s$ and $p_s$ for any contract $m(\cdot,\cdot)$ and expected action $a^e$, this means that the optimal contract must involve payments such that $\int_z \frac{\partial u}{\partial Y}\frac{\partial m}{\partial p_s}dG(z)>0$. *Q.E.D.*

Result A2 generalizes result 4 in the published article. In the absence of managerial trade, the optimal contract set by the shareholders will only vary in the increase in value $V_L-V_s$. However, if the manager can trade then the optimal contract must give the manager an incentive to increase the short-term stock price.

## References

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Laffont, J.-J., 1989, *The Economics of Unvertainty and Information.* Cambridge, Mass.: The MIT Press