Supplemental Materials for Richard Green, "The Electricity Contract Market in England and Wales," *The Journal of Industrial Economics* 47(1), March 1999, pp. 107-124

Appendix A: Risk Aversion

The main paper presents a model in which risk-neutral speculators ensure that the forward price is equal to the expected spot price. This appendix presents a variant in which buyers are risk-averse, and the forward price exceeds the expected spot price by a hedging premium. In practice, contract prices have generally been above the actual Pool prices, and seem to have been above the Pool prices expected at the time when they were signed. Very few speculators have entered the electricity market, and most contracts are bought by suppliers selling on thin margins, who are risk-averse and willing to pay a risk premium. The regulator explicitly allowed National Power and PowerGen to include a hedging premium in their contract prices as part of an agreement to keep wholesale prices below specified levels (Offer, 1994, p v).

We can model the effect of risk aversion if we assume that although the intercept of the demand curve is a known constant when the spot market operates, its value is stochastic and uncertain at the time contracts are signed. This stochastic term has an expected value of A, and variance s^2. Equations (8) and (9) now give the expected values of the spot price and output, respectively.[1] Risk-averse buyers are willing to buy contracts for more than the expected level of

[1] A referee pointed out that if the support of A is large enough, some values would produce zero prices or quantities (under the market rules), and equations (8) and (9) would no longer represent the true expected value. I assume that A is normally distributed, but that its coefficient of variation is sufficiently low that zero prices and quantities would not be observed, and the difference between my equations and the true expected values can be ignored. The model should be viewed as one of competition at times of intermediate demand, and it is almost inconceivable that either prices or the major generators' output would be driven down to zero except at times of very low demand.

the spot price, if they are not fully hedged. A straightforward equation for this, which can be derived from mean-variance utility (Powell, 1993), is:

(A1)
$$f = p^e + l\, s^2\left((q_i^e + q_j^e) - (x_i + x_j)\right)$$

where l is the coefficient of absolute risk aversion and the e superscript denotes an expected value. If we substitute for the expected prices and quantities, we can obtain an equation for *f*:

(A2)
$$f = \frac{1 + 2l\, s^2 b}{2b + b} A - \frac{b + l\, s^2 g}{(2b + b)(b + b)}(x_i + x_j)$$

where $g = (b + b)^2 + b^2$

We can rewrite the firm's expected profits (equation 1) as a function of the two firms' contract sales:

(A3)
$$p_i^e = \mathrm{E}[\, p(x_i, x_j)[q_i(x_i, x_j) - x_i] + f(x_i, x_j)x_i - \tfrac{1}{2} c\, q_i(x_i, x_j)^2 \,]$$

If we differentiate with respect to the firm's contract sales, we get:

(A4)
$$\frac{dp_i^e}{dx_i} = \frac{dp^e}{dx_i}(q_i^e - x_i) + (p^e - cq_i^e)\frac{dq_i^e}{dx_i} + f - p^e + x_i\frac{df}{dx_i}$$
$$+ \frac{dx_j}{dx_i}\left(\frac{dp^e}{dx_j}(q_i^e - x_i) + (p^e - c_i q_i^e)\frac{dq_i^e}{dx_j} + x_i\frac{df}{dx_j}\right)$$

Some more manipulation gives us a first order condition for *x* :

(A5)
$$x_i = \frac{l\, s^2\left(2b(b + b)A - g\, x_j\right) - \frac{dx_j}{dx_i}\frac{b^2}{b + b}\left(A - \frac{b}{b + b}x_j\right)}{l\, s^2 g\left(2 + \frac{dx_j}{dx_i}\right) + b}$$

The second derivative is negative, as required, since each of the terms in square brackets is between zero and one:

(A6)
$$\frac{d^2 p_i^e}{dx_i^2} = -\frac{1}{2b + b}\frac{1}{b + b}\left(2l\, s^2 g\left[1 + \frac{dx_j}{dx_i}\right] + b\left[1 - \frac{b^2}{(b + b)^2}\left(\frac{dx_j}{dx_i}\right)^2\right]\right)$$

An equilibrium in the contract market consists of a pair of contract sales and a pair of conjectural variations that solve equation (A5) for each of the two firms. If the firms have the same conjectural variation, the equilibrium will be symmetric. If the firms have different conjectural variations, there will be asymmetric equilibria. For a symmetric equilibrium, the first part of proposition 2 still holds:

Proposition A1: In a symmetric equilibrium, if both firms have `Bertrand' conjectures ($dx_j / dx_i = -1$), both will sell contracts to cover their expected output, and the expected price in both the contract and the spot market will equal the marginal cost of that output.

Proof: In a symmetric equilibrium, we can eliminate x_j from equation (A5):

$$x_i = \frac{2 l s^2 b(b + b) - \frac{dx_j}{dx_i} \frac{b^2}{b + b}}{l s^2 g \left(3 + \frac{dx_j}{dx_i}\right) + b - \frac{dx_j}{dx_i} \frac{b^3}{(b + b)^2}} A \qquad \text{(A7)}$$

If $dx_j / dx_i = -1$, then we get $x_i = A\, b(b + b)/g$, which implies that $q^e_i = x_i$, and that the spot price will equal marginal cost at this level of output. Since the expected output is fully hedged, equation A1 confirms that there is no hedging premium. QED

The second part of proposition 2 no longer holds once there is a risk premium for contracts, for firms with Cournot conjectures will use the contract market:

Proposition A2: If both firms have `Cournot' conjectures ($dx_j / dx_i = 0$), they will sell contracts for part of their expected output in a symmetric equilibrium, the spot price for this output will be above its marginal cost, and the contract price will be above the expected spot price.

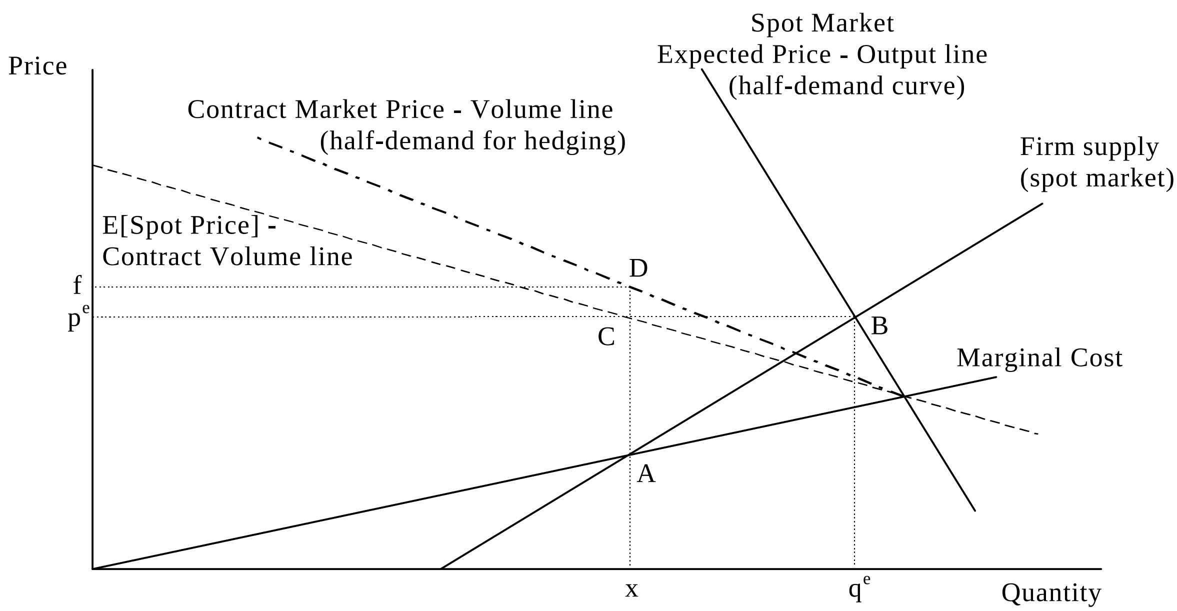
Proof: Equation (A7) shows that the firm sells contracts, but fewer than $A\ b(b + b)/g$. Equation (9) shows that the firm will not be fully hedged if it sells its expected output, proposition 1 that the spot price for this output exceeds marginal cost, and equation (A1) that the contract price exceeds the expected spot price. QED

The discussion of proposition 2 argued that a firm needed a reason, such as changing its opponent's strategy, to enter the contract market. The ability to earn a hedging premium gives another motive for selling contracts, so that a firm will now hedge part of its output, even if this does not affect its rival's strategy and reduces its spot market profits, in order to earn a hedging premium.

As before, conjectural variations of between 0 and -1 will give equilibria between the extremes discussed in the propositions. Figure A1 shows a symmetric equilibrium in which the firms have covered part of their expected output. The supply function in the spot market is the same as in figure 2: the firm has sold contracts for x, and the supply function crosses the marginal cost curve at this level of output, at A. The downwards-sloping solid line from figure 2 now shows the *expected* relationship between the spot price and the firm's output, since demand is stochastic. The expected equilibrium is at B, giving an expected price of p^e and an expected quantity of q^e. The dashed line passing through C shows how the expected price in the spot market is a function of the volume of contracts sold, and is in the same position as the contract market price-volume line, based on arbitrage, in figure 2. With risk-averse buyers, however, the contract price exceeds the expected spot price, and equilibrium in the contract market is found at D. The downwards-sloping dotted and dashed line shows how the contract price depends upon the number of contracts sold. It is half of the industry-wide demand curve for contracts, based upon the hedging equation (A2). The line is drawn to show the equilibria in the contract market and therefore stops before it reaches the vertical axis. This is because the generators will always sell some contracts, even with conjectural variations of zero.

We can quantify the effects of risk aversion if we use the same parameters as before, and assume that $s^2 = 4$, and that $l = 0.03$.[2] The spot market equilibrium is the same as before, so if the generators sell contracts for 92% of their expected output, they can raise the expected spot price to £31/MWh, while reducing their output by 1½%. The RECs are willing to pay a hedging premium of 44p/MWh, or 1.4% of the expected spot price in order to buy in the contract market. The generators' expected revenues are 3% greater than if they sold at marginal cost, which could translate into a 20% difference in their profits.

Figure A1: Symmetric Market Equilibrium
with risk-averse buyers



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[2] This value of l gives relative risk aversion of about 1. The RECs' profits in supply are between 1 and 2% of their supply turnover, or about 3% of their purchase costs (since generation costs are around 60% of turnover). With generation costs of 900, this implies profits of about 30.

Appendix B: Data Sources

The penultimate section of "The Electricity Contract Market in England and Wales", gives information on the contracts sold by National Power and PowerGen, and on the prices which they receive for different types of sales. This appendix provides details of the sources used.

National Power.

TWh	Output	Coal CfDs	Other CfD (net)	Direct Sales
1990/1	121.8	101	6	14
1991/2	117.1	102	0	18
1992/3	108.6	103	5	10
1993/4	94.6	67	13	7
1994/5	92.3	48	27	16
1995/6	88.4	48	19	17

Output

Up to 1994/5 is given in MMC (1996a) Table 4.7

1995/6 is in the National Power Annual Report and Accounts for 1995/6

Coal CfDs

1990/1-1992/3: MMC(1996a) Table 4.10 gives the figures for 1991/2 and 1992/3. For 1990/1, it gives a figure of 122 TWh, which must include contracts intended for the non-franchise market. The volume of contracts assigned to the franchise market did not change significantly in the first three years, and the figure chosen reflects this.

1993/4 - 1995/6: The five-year Coal CfDs covered 67 TWh in their first year, and a total of 193 TWh in the remaining four years. Spreading this evenly gives 48 TWh in each year. (National Power 1993 Annual Report, p. 3).

Other CfD (net)

MMC (1996a) Appendix 5.9 gives the total contract sales to RECs, plus NP's purchases in the contract market. The company bought 15.9 TWh in 1990/1 (as part of the transitional arrangements through which Nuclear Electric provided low-cost electricity resold to large customers who would otherwise have seen price increases in real terms), and 0.1 TWh in 1995/6 (probably to improve their cover when they were expecting to be over-contracted). Given our estimates of the coal contract volumes, the other CfDs are a residual. Note that any errors in the estimates of the coal contract volumes will affect the split between coal and other CfDs, but not the total contract volume.

TWh	Total CfDs	Coal CfDs	Other CfD (gross)	NP purchases	Other CfD (net)
1990/1	122.5	101	21	15.9	6
1991/2	102.0	102	0		0
1992/3	108.2	103	5		5
1993/4	79.6	67	13		13
1994/5	75.4	48	27		27
1995/6	67.3	48	19	0.1	19

Direct Sales to consumers are given in MMC (1996a) Appendix 5.9, and grossed up by 5% to allow for losses in transmission and distribution.

PowerGen

TWh	Output	Coal CfDs	Other CfD (net)	Direct Sales
1990/1	76.1	51.2	16	8
1991/2	75.2	53.0	3	13
1992/3	73.5	50.7	4	18
1993/4	70.2	44.3	4	19
1994/5	70.9	34.0	14	17
1995/6	65.3	34.0	13	19

Output:
Up to 1994/95 is given in MMC (1996b) Table 4.2
1995/96 is in the PowerGen report and accounts for 1996 (page 7).

Coal CfDs: Figures for 1990/1 to 1994/5 are given in MMC (1996b) Table 4.10. The volume in 1995/6 is assumed to be the same as in 1994/5.

Other CfDs: MMC (1996b), Appendix 5.9 gives figures for PowerGen's total CfD cover and its contract purchases for all six years. Deducting the coal CfDs gives a gross figure for the company's other CfDs; deducting purchases gives the net figure. Note that errors in the volume of coal CfDs will lead to corresponding errors in the volume of other contracts, but that the total will remain accurate.

TWh	Total CfDs	Coal CfDs	Other CfD (gross)	PG purchases	Other CfD (net)
1990/1	71.2	51.2	20.0	4.4	15.6
1991/2	57.6	53.0	4.6	1.6	3.0
1992/3	55.8	50.7	5.1	1.5	3.6
1993/4	51.7	44.3	7.4	3.7	3.7
1994/5	51.0	34.0	17.0	2.9	14.1
1995/6	54.1	34.0	20.1	7.3	12.8

Direct Sales are given in MMC (1996b) Table 4.10, scaled up by 5% to account for losses. The figure for 1995/96 is in the PowerGen report and accounts for 1996 (page 7).

Prices:
Figures for 1995/6 are not available. The data in figure 6 have been reflated to October 1992 prices, using the retail price index. The monthly figure for October has been used for the entire financial year, which is the practice of the Office of Electricity Regulation.

Nominal Prices

£/MWh	Pool Purchase Price (time-weighted)	NP-weighted PPP	Coal CfDs	Other NP sales	RPI
1990/1	17.4	18.6	n.a.		130.3
1991/2	20.8	22.0	35.3	22.3	135.6
1992/3	22.8	23.5	34.9	26.0	139.7
1993/4	24.4	25.1	32.7	27.1	141.6
1994/5	24.0	27.6	33.4	27.3	145.3

October 1992 Prices

£/MWh	Pool Purchase Price (time-weighted)	NP-weighted PPP	Coal CfDs	Other NP sales
1990/1	18.7	19.9	n.a.	
1991/2	21.4	22.7	36.3	23.0
1992/3	22.8	23.5	34.9	26.0
1993/4	24.1	24.8	32.2	26.7
1994/5	23.1	26.5	32.1	26.3

MMC (1996a) table 4.8 gives annual average figures for the time-weighted Pool Purchase Price and the Pool Purchase Price weighted by National Power's output.

Coal CfD prices were calculated by dividing National Power's income related to those contracts (MMC, 1996a, table 4.12) by the amount of electricity covered (discussed above). The figures in October 1992 prices can be compared with those quoted by Eastern Electricity for a standardised demand pattern (Trade and Industry Committee, 1992, page 63): £32.6/MWh in 1993/4 and £32.2/MWh in 1994/5.

£m	Pool income	Coal support	Contract payments (net)	Total	£/MWh
1990/1	n.a.	611	n.a.		
1991/2	2262	477	857	3596	35.3
1992/3	2383	297	911	3591	34.9
1993/4	1666	132	392	2190	32.7
1994/5	1368	79	163	1610	33.4

Other sales
MMC (1996a) table 4.9 gives National Power's difference payments from its non-coal CfDs. We assume that these include the CfDs with its direct sales business, and divide the payments by the

relevant volume to get an amount per MWh. This is added to National Power's average Pool Purchase Price to get a price for the contracts.

	CfD payments £m	Volume TWh	Premium £/MWh	NP's PPP	Total
1990/1	n.a.		n.a.	18.6	n.a.
1991/2	6	17.8	0.3	22.0	22.3
1992/3	38	15.0	2.5	23.5	26.0
1993/4	40	19.8	2.0	25.1	27.1
1994/5	-12	43.5	-0.3	27.6	27.3

Contracts for Coal

Trade and Industry Committee (1993) para 34 gives British Coal's price in 1992/3 (£1.86 per GJ), and the expected prices in 1993/4 (£1.51 per GJ) and 1997/8 (£1.33 per GJ). The volume in 1990/1 and 1991/2 was 70 million tonnes (para 21), falling to 65 million tonnes in 1992/3. The report correctly anticipates the volumes in the second set of contracts as 40 million tonnes in 1993/4 and 30 million tonnes in the following years (para 29).

National Power's Operating Profits from Coal Contracts

MMC (1996a) Table 4.12 gives National Power's attribution of revenues and costs to the coal-backed contracts, and derives figures for the "gross margin". The margin per MWh is derived from these figures.

	Gross margin £m	Volume TWh	Margin £/MWh
1990/1	n.a.	101	n.a.
1991/2	783	102	7.7
1992/3	900	103	8.7
1993/4	649	67	9.7
1994/5	621	48	12.9

References

Monopolies and Mergers Commission (1996a) *National Power PLC and Southern Electric plc: A report on the proposed merger,* Cm 3230, London, HMSO

Monopolies and Mergers Commission (1996b) *PowerGen plc and Midlands Electricity plc: A report on the proposed merger,* Cm 3231, London, HMSO

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