Supplemental Materials for D. Lee Heavner, "Vertical Enclosure: Vertical Integration and the Reluctance to Purchase from a Competitor," *The Journal of Industrial Economics,* LII (2), June 2004, pp. 179-199

Appendix A: Discussion of Counterintegration

Allowing $U2$ and $D2$ to integrate does not affect the model's prediction that enclosure costs can make it unprofitable for a technologically superior upstream unit to integrate downstream. To focus on this result, the appendix considers the case where $U1$ has a technological advantage (i.e., $\Delta \geq 0$).

To incorporate $U2$ and $D2$'s ability to integrate, I modify the date zero stage of the model as follows. At date zero, $U1$ and $D1$ decide whether to integrate. After observing $U1$ and $D1$'s organizational form, $U2$ and $D2$ decide whether to integrate. $U2$ and $D2$ employ the same tie-breaking rule as $U1$ and $D1$ in that $U2$ and $D2$ integrate whenever they are indifferent between integrating and not integrating.[1]

Because the downstream units are identical at date zero, the gains from vertical integration do not depend on the identity of the integrating downstream unit. Thus, I assume, without loss of generality, that if Ui integrates, then Ui integrates with Di for $i = 1, 2$.

If $U1$ has a technological advantage, then integration is a weakly dominant strategy for $U2$ and $D2$. To see that this is true, note that given $U1$ and $D1$'s organizational form, the following statements are true: i) $U2$-$D2$ integration increases the profitability of $U2$-$D2$ trade; ii) $U2$-$D2$ integration does not affect any unit's expected gain from $U1$-$D2$ trade; and

iii) $U1$'s technological advantage leads $D1$ to purchase from $U1$ regardless of $U2$ and $D2$'s organizational form. Hence, $U2$-$D2$ integration cannot decrease $U2$ and $D2$'s joint payoff. Given this result, it is straightforward to prove the following.

Proposition 4. *There exists a range of technologies, $\Delta \in [X, V)$ such that i) $X > 0$; ii) $D2$ purchases from $U1$ if and only if $U1$ and $D1$ do not integrate, and iii) $U1$ and $D1$ do not integrate.*

Appendix B: Proofs

Proof to Lemma 1.

Let π^i and π_j denote unit Ui's and Dj's respective payoffs. Solving the optimization in (3) shows that if $U1$, $D1$, $U2$, and $D2$ are independent, then the units earn the following expected payoffs

$$\begin{aligned} \pi^1 &= (I_1 + I_2)\left(\tfrac{\Delta + f(h(2))}{2} - h(2)\right) \\ \pi^2 &= (2 - I_1 - I_2)\left(\tfrac{f(h(2))}{2} - h(2)\right) \\ \pi_i &= k_i + \left(\tfrac{I_i}{2} - I_j\gamma\right)\Delta + \left(\tfrac{1}{2} - \gamma\right) f(h(2)), \text{ for } i, j = 1, 2;\ i \neq j \end{aligned} \tag{5}$$

Straightforward comparisons of these payoffs completes the proof.

Proof to Lemma 2.

Let π_1^1 denotes an integrated $U1$-$D1$'s expected payoff. Solving the optimization in (4) shows that if $U1$ and $D1$ are integrated and if $U2$ and $D2$ are independent, then the units earn the following expected payoffs.

$$
\begin{aligned}
\pi_1^1 &= \begin{array}{c} k_1 + I_1\left[\Delta + f\left(h\left(1\right)\right) - h\left(1\right)\right] + I_2\left(\frac{1}{2}\left(1-\gamma\right)\left(\Delta + f\left(h\left(\frac{2}{1-\gamma}\right)\right)\right) - h\left(\frac{2}{1-\gamma}\right)\right) \\ + \left(\frac{1-I_1}{2} - \left(1-I_2\right)\gamma\right) f\left(h\left(2\right)\right) \end{array} \\
\pi^2 &= \left(2 - I_1 - I_2\right)\left(\frac{f(h(2))}{2} - h\left(2\right)\right) \qquad (6)\\
\pi_2 &= \begin{array}{c} k_2 + I_2\frac{1}{2}\left(1-\gamma\right)\left(\Delta + f\left(h\left(\frac{2}{1-\gamma}\right)\right)\right) + \left(1-I_2\right)\frac{f(h(2))}{2} \\ -\gamma\left[I_1\left(\Delta + f\left(h\left(1\right)\right)\right) - \left(1-I_1\right) f\left(h\left(2\right)\right)\right] \end{array}
\end{aligned}
$$

Part (i). Define W and $Y(\gamma)$ as follows.

$$
\begin{aligned}
W &\equiv \frac{f(h(2))}{2} - f\left(h\left(1\right)\right) + h\left(1\right) \\
Y\left(\gamma\right) &\equiv \frac{f(h(2))}{1-\gamma} - f\left(h\left(\frac{2}{1-\gamma}\right)\right)
\end{aligned}
\qquad (7)
$$

The regularity assumptions on f imply $W < 0 < Y(\gamma)$.

Part (ii) - (iv). Comparing the payoffs in (6) shows that a) $U2$ always prefers to trade with as many downstream units as possible; b) if $\Delta \geq Y(\gamma)$, then $D2$ prefers to trade with $U1$; c) if $\Delta < Y(\gamma)$, then $D2$ prefers to trade with $U2$; d) if $\Delta \geq W$, then $U1$ and $D1$ trade internally, and e) if $\Delta < W$, then $D1$ orders from $U2$.

$U1$ prefers to invest in supplying quality to $D2$ rather than having $D2$ order from $U2$ if and only if

$$
\Delta > -f\left(h\left(\frac{2}{1-\gamma}\right)\right) + \frac{2}{1-\gamma}\left(h\left(\frac{2}{1-\gamma}\right) - \gamma f\left(h\left(2\right)\right)\right) \qquad (8)
$$

However, $U1$'s incentive to invest in $D2$'s quality is lower after $D2$ commits to purchasing from $U1$ (and commits to not purchasing from $U2$). If $D2$ has committed to trading with

$U1$, then $U1$ will invest in supplying quality to $D2$ if and only if

$$\Delta > -f\left(h\left(\frac{2}{1-\gamma}\right)\right) + \frac{2}{1-\gamma}h\left(\frac{2}{1-\gamma}\right) \tag{9}$$

The regularity conditions on f imply that the right side of the inequalities in (8) and (9) are strictly less than $Y(\gamma)$; thus, if $D2$ prefers to order from $U1$-$D1$, then $U1$-$D1$ will invest in supplying quality to $D2$. Hence, statements (a)-(e) determine the equilibrium order placements. Statements (a)-(e) also prove that W and $Y(\gamma)$ satisfy parts (ii) - (iv) of the lemma.

Straightforward substitution proves parts $Y(0) = 0$ and $\lim_{\gamma\to 1} Y(\gamma) = \infty$. Differentiating $Y(\gamma)$ gives

$$\frac{dY(\gamma)}{d\gamma} = \frac{f(h(2))}{(1-\gamma)^2} - \frac{4}{(1-\gamma)^3}\frac{dh\left(\frac{2}{1-\gamma}\right)}{d\gamma}$$

The regularity conditions on f make the first term of this derivative positive. The concavity of f and the inverse function properties of h imply that $h(x)$ is decreasing in x. Thus, $\frac{dh\left(\frac{2}{1-\gamma}\right)}{d\gamma} < 0$ for $\gamma \in (0,1)$. Therefore, $\frac{dY(\gamma)}{d\gamma} > 0$ for all $\gamma \in (0,1)$. □

Proof to Proposition 2.

Let B denote the bilateral gains from integration. Formally,

$$B \equiv f(h(1)) - h(1) - f(h(2)) + h(2) \tag{10}$$

Part (i) follows immediately from the lemmas.

The lemmas show that $\Delta \geq Y$ implies that $D1$ and $D2$ order from $U1$ regardless of

$U1$ and $D1$'s integration decision. Comparing (5) and (6) shows that if both $D1$ and $D2$ purchase from $U1$, then integration (weakly) increases $U1$ and $D1$'s joint payoff if and only if

$$(11) \quad \left(\frac{1}{2}-\gamma\right) f\left(h\left(2\right)\right) - h\left(2\right) - \left[\frac{1-\gamma}{2} f\left(h\left(\frac{2}{1-\gamma}\right)\right) - h\left(\frac{2}{1-\gamma}\right)\right] \leq \frac{\gamma\Delta}{2} + B$$

The regularity conditions on f imply that B is positive and that

$$\frac{1-\gamma}{2} f\left(h\left(2\right)\right) - h\left(2\right) < \frac{1-\gamma}{2} f\left(h\left(\frac{2}{1-\gamma}\right)\right) - h\left(\frac{2}{1-\gamma}\right)$$

Thus, (11) holds for all nonnegative Δ. $Y(\gamma) > 0$, so $\Delta \geq Y(\gamma)$ implies that $U1$-$D1$ integration is profitable. This proves part (ii).

The lemmas show that $\Delta \in [W, 0)$ implies the following: a) If $U1$ and $D1$ integrate, then $D1$ orders from $U1$, and $D2$ orders from $U2$. b) If $U1$ and $D1$ do not integrate, then both $D1$ and $D2$ order from $U2$. Comparing (5) and (6) and using (7) shows that if $\Delta \in [W, 0)$, then $U1$ and $D1$ earn a larger joint payoff from outcome (a) than from outcome (b). Thus, if $\Delta \in [W, 0)$, then $U1$ and $D1$ integrate; $D1$ orders from $U1$, and $D2$ orders from $U2$. This proves part (iii). □

Proof of proposition 3.

Lemmas 1 and 2 say that $\Delta \in [0, Y(\gamma))$ implies a) $D1$ purchases from $U1$; b) if $U1$ and $D1$ do not integrate, then $D2$ purchases from $U1$, and c) if $U1$ and $D1$ integrate, then $D2$ purchases from $U2$. Given these trading strategies, $U1$ and $D1$ will integrate if and only if

$$(12) \quad \pi_1^1 \geq \pi^1 + \pi_1$$

Define $B^*(\Delta,\gamma)$ as follows

$$B^*(\Delta,\gamma) \equiv \frac{1-2\gamma}{2}\Delta + \frac{1}{2}f(h(2)) - h(2)$$

Using the payoffs in (6) and (5) shows that (12) is equivalent to

$$B \geq B^*(\Delta,\gamma)$$

Straightforward calculations show the following:

$$\begin{gathered} B^*(0,\gamma) > 0 \\ \frac{\partial B^*(0,\gamma)}{\partial\gamma} = 0;\ \frac{\partial B^*(\Delta,\gamma)}{\partial\gamma} < 0, \text{ for } \Delta > 0 \\ \frac{\partial B^*(\Delta,\gamma)}{\partial\Delta} > 0, \text{ for } \gamma < \tfrac{1}{2};\ \frac{\partial B^*(\Delta,\gamma)}{\partial\Delta} < 0, \text{ for } \gamma > \tfrac{1}{2} \end{gathered} \tag{13}$$

Proof to Proposition 4.

Step 1. Assume $\Delta \geq 0$. Appendix A shows that $\Delta \geq 0$ implies that $U2$ and $D2$ integrate.

Define X and Z as follows

$$\begin{gathered} X \equiv 2f(h(1)) - 2h(1) - f(h(2)) \\ Z(\gamma) \equiv \tfrac{2}{1-\gamma}\left(f(h(1)) - h(1)\right) - f\left(h\left(\tfrac{2}{1-\gamma}\right)\right) \end{gathered}$$

The regularity conditions on f imply

$$f(h(1)) - h(1) > f(h(2)) - h(2) \tag{14}$$

and

$$f(h(2)) > 2h(2) \tag{15}$$

Combining (14) and (15) shows that $X > 0$. Comparing X and Z shows that $X < Z$ for all $\gamma \in (0, 1)$.

Straightforward calculations similar to the calculations used to prove the lemmas show the following: a) $\Delta \geq 0$ implies that $D1$ purchases from $U1$ regardless of $U1$ and $D1$'s integration decision. b) The regularity conditions on f and $\Delta \geq 0$ imply that if $D2$ orders from $U1$, then $U1$ sells to $D2$ and invest a positive amount in $D2$'s input quality regardless of whether $U1$ and $D1$ integrate. c) If $U1$ and $D1$ do not integrate, then an integrated $U2$-$D2$ purchases from $U1$ if $\Delta \geq X$ and an integrated $U2$-$D2$ trades internally if $\Delta < X$. d) If $U1$ and $D1$ are integrated, then an integrated $U2$-$D2$ purchases from $U1$ if $\Delta > Z$ and an integrated $U2$-$D2$ trades internally if $\Delta < Z$. Thus $\Delta \in [X, Z)$ is a range of technological advantages for which an integrated $U2$-$D2$ purchases from $U1$ if and only if $U1$ and $D1$ are not integrated.

Step 2. If $\Delta \in [X, Z)$, then $D2$'s purchasing strategy implies that $U1$ and $D1$ are better off remaining independent if and only if

$$\left(\frac{1}{2} - \gamma\right)\Delta > (1 - \gamma) f(h(1)) - h(1) - \left(\frac{3}{2} - \gamma\right) f(h(2)) + 2h(2)$$

At $\Delta = X + 2\varepsilon$, this inequality is equivalent to

$$\varepsilon - \gamma (f(h(1)) - 2h(1) + 2\varepsilon) > -f(h(2)) + 2h(2) \tag{16}$$

The left side of (16) is linear in γ, so if (16) holds for $\gamma = 0$ and $\gamma = 1$, then (16) holds for all $\gamma \in [0, 1]$. At $\gamma = 0$, (16) is equivalent to

$$\varepsilon > -f(h(2)) + 2h(2)$$

The regularity conditions on f imply that this inequality holds for $\varepsilon \geq 0$. At $\gamma = 1$, (16) is equivalent to

$$\varepsilon < f(h(2)) - 2h(2) - f(h(1)) + 2h(1) \tag{17}$$

The regularity conditions on f imply that the right side of (17) is positive, so there exists an $\varepsilon_C > 0$ such that (17) holds for all $\varepsilon \in [0, \varepsilon_C)$.

Define

$$V \equiv \min\{Z, X + 2\varepsilon_C\}$$

From above, if $\Delta \in [X, V)$, then $U1$-$D1$ integration reduces the sum of $U1$'s and $D1$'s payoffs for all $\gamma \in (0, 1)$. Hence, technologies with $\Delta \in [X, V)$ satisfy part (iii) of the proposition. Step 1 of the proof and the definition of V shows that these technologies also satisfy parts (i) and (ii) of the proposition. □

Notes

[1]There are other ways to model vertical integration decisions within a successive duopoly, and the modelling assumptions can affect the equilibrium integration decisions. For this

reason, I do not claim to predict the equilibrium industry structure. Instead, I am content to show that allowing $U2$-$D2$ integration does not invalidate the claim that enclosure costs can lead firms to forgo vertical integration.