# THE EFFECT OF AUCTION FORMAT ON EFFICIENCY AND REVENUE IN DIVISIBLE GOODS AUCTIONS: A TEST USING KOREAN TREASURY AUCTIONS[*]

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This paper measures the efficiency and revenue properties of the two most popular formats for divisible goods auctions – the uniform-price and discriminatory auction. We analyze bids into the Korean Treasury auctions which have used both formats. We find that the discriminatory auction yields statistically higher revenue. Unlike previous work that uses data from only one format, we are able to compare the efficiency properties of the two formats. We find that the discriminatory auction better allocates treasury bills to the highest value financial institutions. However, the differences in revenue and efficiency are not large because the auctions are very competitive.

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## SUPPLEMENTARY SECTION (for our website): Derivation of the FOCs for the Continuous Case and Comparison of Results for Continuous vs. Discrete FOCs

*1. Discriminatory Auction*

The bidder's expected profit maximization problem is:

(1) $$\max_{y(.)} \int_0^\infty \{\int_0^{y(p^c)} [v(q,t_i) - y^{-1}(q)]\,dq\}\,dH(p^c, y(p^c))$$

Note that we drop the *i* subscript from the demand function.

Let the term in the { } be $\pi(y(p))$, the surplus from winning $y(p)$. Then

$$\frac{d\pi}{dp} = \frac{d\pi}{dq}\cdot\frac{dq}{dp} = [v(y(p),t_i) - y^{-1}(y(p))]\,y'(p)$$

Integrating by parts, we get:

$$\int_0^\infty \pi(y(p))\,dH(p,y(p)) = \pi(y(p))\,H(p,y(p))\Big|_0^\infty - \int_0^\infty H(p,y(p))\,d\pi(y(p))$$

If $p=\infty$ then $\pi(y(\infty))=0$, and if $p=0$ then $H(0)=0$. Equation (1) becomes:

(2) $$\max_{y_i(.)} -\int_0^\infty H(p,y(p))[v(y(p),t_i) - p]\,y'(p)\,dp$$

Observe that the integrand is a function of *p, y* and *y'*, denote it by *F(p,y,y')*. The Euler equation which is a necessary condition for optimality is given by $F_y = \frac{d}{dp}F_{y'}$.

Therefore, the Euler condition for the differential equation (2) is given by

(3) $$v(y_i(p),t_i) = p + \frac{H(p,y_i(p))}{H_p(p,y_i(p))}$$

*2. Uniform Price Auction*

The bidder's expected profit maximization problem is:

(4) $$\max_{x_i(.)} \int_0^\infty \{\int_0^{x(p)} v(q,t_i)\,dq - px(p)\}\,dG(p,x(p))$$

Let the term in { } be $\pi(x(p))$ which is surplus from winning $x(p)$ units. Then

$$\frac{d\pi}{dp} = [v(x(p),t_i) - p]x'(p) - x(p)$$

Using the same boundary conditions and integration by parts, (4) becomes

(5) $$\max_{x(.)} -\int_0^\infty G(p, x(p))\{[v(x(p), t_i) - p]x'(p) - x(p)\}\,dp$$

After solving equation (5), we get (6) as the Euler condition for the uniform price auction.

(6) $$v(x_i(p), t_i) = p - \frac{G_x(p, x_i(p))}{G_p(p, x_i(p))} \cdot x(p)$$

*3. Empirical Results*

For comparison, we present results for both the discrete and continuous FOC. The results can be different, and it is these differences that lead us to emphasize the discrete results in the main text. The discrete FOC best represents the actual bidding in the Korean Treasury auctions.

| | | % Efficiency Loss | | % Revenue Loss | |
|---|---|---|---|---|---|
| **Format** | **Date** | **Discrete** | **Continuous** | **Discrete** | **Continuous** |
| Discriminatory | 9/13/1999 | 0.006 | 0.000% | 0.100 | 0.051% |
| Discriminatory | 10/11/1999 | 0.000 | 0.001% | 0.033 | 0.020% |
| Discriminatory | 11/15/1999 | 0.000 | 0.001% | 0.023 | 0.005% |
| Discriminatory | 1/17/2000 | 0.000 | 0.001% | 0.039 | 0.033% |
| Discriminatory | 2/14/2000 | 0.000 | 0.000% | 0.029 | 0.025% |
| Discriminatory | 3/13/2000 | 0.000 | 0.000% | 0.027 | 0.020% |
| Discriminatory | 4/10/2000 | 0.000 | 0.001% | 0.018 | 0.010% |
| Discriminatory | 5/8/2000 | 0.000 | 0.000% | 0.017 | 0.006% |
| Discriminatory | 6/12/2000 | 0.004 | 0.001% | 0.071 | 0.041% |
| Discriminatory | 7/10/2000 | 0.009 | 0.000% | 0.051 | 0.034% |
| Discriminatory | **Mean** | **0.002** | **0.001%** | **0.041** | **0.025%** |
| Uniform-Price | 8/14/2000 | 0.018 | 0.000% | -0.027 | -0.002% |
| Uniform-Price | 9/18/2000 | 0.002 | 0.001% | -0.002 | -0.012% |
| Uniform-Price | 10/9/2000 | 0.038 | 0.000% | -0.027 | -0.004% |
| Uniform-Price | 11/13/2000 | 0.056 | 0.000% | -0.027 | -0.001% |
| Uniform-Price | 1/8/2001 | 0.078 | 0.000% | -0.031 | -0.002% |
| Uniform-Price | 2/5/2001 | 0.025 | 0.000% | -0.028 | -0.008% |
| Uniform-Price | 3/12/2001 | 0.038 | 0.000% | -0.027 | -0.002% |
| Uniform-Price | 4/2/2001 | 0.312 | 0.000% | -0.037 | -0.008% |
| Uniform-Price | 5/7/2001 | 0.018 | 0.000% | -0.040 | -0.009% |
| Uniform-Price | 6/4/2001 | 0.025 | 0.008% | -0.027 | -0.032% |
| Uniform-Price | 7/2/2001 | 0.001 | 0.001% | -0.025 | -0.012% |
| Uniform-Price | 8/6/2001 | 0.021 | 0.000% | -0.028 | -0.021% |
| Uniform-Price | 9/3/2001 | 0.026 | 0.000% | -0.028 | -0.004% |
| Uniform-Price | 10/8/2001 | 0.020 | 0.000% | -0.022 | -0.003% |
| Uniform-Price | 11/7/2001 | 0.011 | 0.000% | -0.012 | -0.003% |
| Uniform-Price | 12/3/2001 | 0.060 | 0.000% | -0.028 | -0.007% |
| Uniform-Price | 1/7/2002 | 0.009 | 0.028% | -0.028 | -0.005% |
| Uniform-Price | 2/4/2002 | 0.014 | 0.001% | -0.027 | -0.008% |
| Uniform-Price | 3/4/2002 | 0.031 | 0.000% | -0.034 | -0.008% |
| Uniform-Price | 4/1/2002 | 0.034 | 0.000% | -0.027 | -0.004% |
| Uniform-Price | **Mean** | **0.042** | **0.002%** | **-0.027** | **-0.008%** |