Supplemental Materials for David A. Malueg and Marius Schwartz, “International Telecom Settlements: Gaming Incentives, Carrier Alliances, and Pareto-Superior Reform,” *The Journal of Industrial Economics* VOLUME (ISSUE), MONTH YEAR, pp. XXX–YYY.

Marginal Cost under Alternative Expectations

Recall the generalized expression for marginal cost:

$$\text{(3)} \qquad \frac{d(C_j - c_d n_j)}{dn_j} = AC + (r - c_d)\frac{N_I}{N_O} n_j \left(\frac{dN_O / dn_j}{N_O} - \frac{dN_I / dn_j}{N_I} \right).$$

Properties (a), (b) and (c) of *MC* described in the text will hold if (not only if) in (3) the expected term

$$\frac{dN_O / dn_j}{N_O} - \frac{dN_I / dn_j}{N_I}$$

is the same across carriers and is positive. Suppose that outbound traffic does not affect inbound traffic: $dN_I/dn_j = 0$. Then if each carrier *j* expects $dN_O/dn_j = 0$, carriers’ marginal costs will *not* depend on their initial market shares. This case of pure output diversion, $dN_O/dn_j = 0$, however, is extreme. For example, if carriers’ products are differentiated and carrier *j* offers a price cut to all customers while others’ prices are constant (as under Bertrand competition), a more likely expectation is some mix of diversion and industry expansion, $0 < dN_O/dn_j < 1$. If this expectation is the same across carriers (e.g., for linear differentiated demands and different intercepts but symmetric cross partials), then—continuing to assume N_I constant—properties (a) through (c) would hold.[1]

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[1] To the extent carriers might expect different ratios of diversion to expansion when they increase their outputs, the expected dN_O/dn_j arguably should be greater for larger than for smaller carriers, because a lesser portion of their expansion is likely to come from diversion (since larger carriers face a smaller

To understand why the pure diversion case is special, suppose that as any firm j expands traffic by t, total outbound traffic changes by ΔN_O (instead of by t as in (5)). The change in j's share of traffic can be written as

$$\Delta s_j = \frac{n_j + t}{N_O + \Delta N_O} - \frac{n_j}{N_O} = \frac{t}{N_O} - \left(\frac{1}{N_O} - \frac{1}{N_O + \Delta N_O}\right)(n_j + t).$$

The term t/N_O is the increase in share that would have resulted if the t minutes were diverted from other carriers, so total outbound traffic remained unchanged. But after total traffic increases, one minute represents a market share of $1/(N_O + \Delta N_O)$ instead of $1/N_O$. This discount equal to $1/N_O - 1/(N_O + \Delta N_O)$ in the "share value" of an outbound minute must be applied to all of j's traffic at the new level, $n_j + t$, to arrive at the actual change in j's share; hence the resulting increase in share is smaller the larger is the initial n_j. This effect disappears in the case of pure diversion because there is then no expansion in industry output, hence no discount effect.

We do not mean to push this discussion of expectations too far, but only to suggest that the properties of our base case (4) (other than the linearity of marginal cost in shares) are fairly robust to alternative assumptions about expectations: as can be shown from (3), our properties hold if $d(N_O/N_I)/dn_j$ is positive and is non-decreasing across carriers the larger is a carrier's initial share. The above discussion suggests that these conditions are plausible.

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customer pool from which to divert business). This would preserve property (a), that marginal cost increases with share (although the relation will now be strictly convex rather than linear).

Proof of Proposition 4

Recall from (1) that the cost to carrier *a* of terminating its traffic abroad net of profit from inbound termination is equal to

$$C_a(n_a, N_I) = (r + c_d)n_a - (r - c_d)n_a \frac{N_I}{N_O}.$$

Carrier *a*'s marginal cost of terminating these additional minutes abroad is equal to

(A1) $$MC_a(n_a, N_I) \equiv \frac{\partial C_a}{\partial n_a} = (r + c_d) - (r - c_d)n\frac{N_I}{N_O^2}.$$

(Note: MC_a is *not* "net" of origination costs, as assumed in (3), and therefore is not identical to (3).) Now suppose carriers *f* and *a* arrange to turn around *t* inbound minutes into an equal number of outbound minutes carried by *a*. This reduces total inbound minutes to $N_I - t$ and increases *a*'s outbound minutes to $n_a + t$. With the reduction in inbound minutes, *a*'s marginal cost function, given other firms' minutes, is $MC_a(n_a, N_I - t)$ (recall that this marginal cost curve incorporates the effect that increasing output has in increasing a firm's share of inbound minutes). The change in *a*'s termination cost is given by

$$\begin{aligned} C_a(n_a + t, N_I - t) - C_a(n_a, N_I) &= C_a(n_a + t, N_I) - C_a(n_a, N_I - t) \\ &\quad + C_a(n_a, N_I - t) - C_a(n_a, N_I) \\ &= \int_{n_a}^{n_a + t} MC_a(x, N_I - t)\, dx \\ &\quad + \left((r + c_d)n_a - (r - c_d)n_a \frac{N_I - t}{N_O} \right) \\ &\quad - \left((r + c_d)n_a - (r - c_d)n_a \frac{N_I}{N_O} \right) \\ &= \int_{n_a}^{n_a + t} MC_a(x, N_I - t)\, dx \; + \; (r - c_d)\frac{n_a}{N_O}t. \end{aligned} \qquad \text{(A.2)}$$

In the last line of (A.2), $(r-c_d)t(n_a/N_O)$ represents *a*'s lost profits from the reduction in total inbound minutes ($\Delta N_I = -t < 0$), which caused *a* to lose inbound minutes in proportion to its initial market share of outbound minutes ($s_a = n_a/N_O$). The first term corresponds to the area under the new marginal cost curve (shifted up by the decrease in inbound minutes) between *a*'s old and new levels of outbound minutes. This integral term in (A.2) can be rewritten as follows:

(A.3)

$$\begin{aligned}
\int_{n_a}^{n_a+t} MC_a(x, N_I - t)\, dx &= \int_{n_a}^{n_a+t} \left((r+c_d) - (r-c_d)(N_I - t)n(n+x)^{-2}\right) dx \\
&= \left((r+c_d)x + (r-c_d)(N_I - t)n(n+x)^{-1}\right)\Big|_{n_a}^{n_a+t} \\
&= \left((r+c_d)t + (r-c_d)(N_I - t)n\left(\frac{1}{n+n_a+t} - \frac{1}{n+n_a}\right)\right) \\
&= \left((r+c_d)t + (r-c_d)(N_I - t)n\left(\frac{1}{N_O+t} - \frac{1}{N_O}\right)\right) \\
&= \left((r+c_d)t + (r-c_d)(N_I - t)n\left(\frac{t}{(N_O+t)N_O}\right)\right) \\
&= \left((r+c_d) + (r-c_d)\left(\frac{(N_I - t)n}{(N_O+t)N_O}\right)\right)t \\
&= \left((r+c_d) + (r-c_d)\left(\frac{(N_I - t)n}{(N_O+t)N_O}\right)\right)t \\
&= \left((r+c_d) + (r-c_d)\frac{(N_I - t)}{(N_O+t)N_O}(1-s_a)\right)t.
\end{aligned}$$

In our experiment, for each of the *t* additional outbound minutes that *a* carries due to the turnaround, *a* is paid by *f* a price equal to *a's* new marginal cost of foreign termination. Thus, *a* receives

(A.4) $$MC(n_a+t, N_I-t)\times t = \left((r+c_d)-(r-c_d)\frac{n(N_I-t)}{(N_O+t)^2}\right)t\,.$$

Substituting (A.3) into (A.2) to obtain the increase in *a*'s termination costs, and using (A.4), we can now express the change in *a*'s net revenues from this arrangement as follows:

$$\begin{aligned}
\Delta(\text{net revenue}) &= \text{ payment received } - \text{ increased costs} \\
&= \left((r+c_d)-(r-c_d)\frac{n(N_I-t)}{(N_O+t)^2}\right)t \\
&\quad -\left\{\left((r+c_d)-(r-c_d)\frac{n(N_I-t)}{N_O(N_O+t)}\right)t+(r-c_d)\left(\frac{n_a}{N_O}\right)t\right\} \\
&= (r-c_d)\left(\frac{n(N_I-t)}{(N_O+t)}\right)\left(\frac{1}{N_O}-\frac{1}{(N_O+t)}\right)t-(r-c_d)s_a t \\
\text{(A.5)}\qquad &= (r-c_d)\left(\frac{n(N_I-t)}{(N_O+t)}\right)\left(\frac{t}{N_O(N_O+t)}\right)t-(r-c_d)s_a t \\
&= (r-c_d)\left(\frac{n(N_I-t)t}{N_O(N_O+t)^2}\right)t-(r-c_d)s_a t \\
&= (r-c_d)\left((1-s_a)\frac{(N_I-t)t}{(N_O+t)^2}\right)t-(r-c_d)s_a t \\
&= (r-c_d)t\left[\left(\frac{N_I-t}{N_O+t}\right)\left(\frac{t}{N_O+t}\right)-s_a\left(1+\left(\frac{(N_I-t)t}{(N_O+t)^2}\right)\right)\right] \\
&= (r-c_d)\left[y-s_a(1+y)\right]t\,,
\end{aligned}$$

where $y=\dfrac{(N_I-t)t}{(N_O+t)^2}$.

Expression (A.5) implies that, for given levels of initial traffic and for given turnaround, t, the profitability to carrier a is decreasing linearly in its initial market share of outbound minutes, s_a. It also demonstrates the properties argued informally earlier in the text, that under the hypothesized payment (price equal to the new marginal cost), turnaround will be profitable for a small carrier but not for a large one. For example, for $s_a = 0$, we have $\Delta(\text{net revenue}) = (r - c_d)yt > 0$; but for $s_a = 1$, we have $\Delta(\text{net revenue}) = -(r - c_d)t < 0$. (By similar logic, it is not profitable under the hypothesized payments to turn around all of the minutes, which is seen by noting that such complete turnaround makes $y = 0$, and, hence, $\Delta(\text{net revenue}) = -(r - c_d)s_a t < 0$.)

From (A.5) the critical initial share above which turnaround of t minutes is *not* profitable under the hypothesized reimbursement scheme is

$$s^* = \frac{y}{1+y} = \frac{(N_I - t)t}{(N_O + t)^2 + (N_I - t)t} = \frac{\left(1 - \frac{t}{N_I}\right)\frac{t}{N_I}}{\left(\frac{N_O}{N_I} + \frac{t}{N_I}\right)^2 + \left(1 - \frac{t}{N_I}\right)\frac{t}{N_I}}.$$

For given N_O and N_I, the largest value of s^* is found when $t = \frac{N_O N_I}{2N_O + N_I}$, which implies

$$s^*_{\max} = \left(\frac{N_I}{2N_O + N_I}\right)^2.$$

Observe that if $N_O \geq N_I$, then $s^*_{\max} \leq 1/9 \approx 11\%$, as reported in Proposition 4. *Q.E.D.*