Supplementary Materials for Richard Schmalensee, “Payment Systems and Interchange Fees, *The Journal of Industrial Economics*, Vol. L (2), June 2002, pp. 103-122.

This supplementary appendix generalizes the linear demand system analyzed in Section IV of the text to allow partial demands to exhibit network effects directly, so that the state of the other side of the system affects pricing, and analyzes the relation between maximizing output and maximizing private value. Consider one side of the system with N firms and with demands given by a simple generalization of (10):

$$q_i = Q^{oe}\left\{\frac{A}{N} + \frac{\varphi Q^{oe}}{N} + \Theta\left[\sum_{j\neq i} P_j - (N-1)P_i\right] - \frac{B}{N}P_i\right\},\ i = 1,\ldots,N, \tag{S1}$$

where Q^{oe} is the expected partial demand on the other side of the system, and φ is a positive constant. (Superscripts are generally dropped in this paragraph to avoid clutter.) The larger is φ, the more important are direct network effects on this side of the system. Suppose all firms on this side of the system have unit cost C', net of interchange. (If this is the acquiring side of the system, $C'=C+T$, while $C'=C-T$ on the issuing side.) Multiplying (S1) by (P_i-C'), differentiating, and solving for a symmetric equilibrium conditional on Q^{oe}, we obtain

$$Q = \frac{\left(A + \varphi Q^{oe} - BC'\right)\left(B + \beta\right)}{(2B + \beta)}, \tag{S2}$$

where, as in the text, $\beta = N(N-1)\Theta$.

To obtain a symmetric fulfilled expectations equilibrium, one must solve for conditional equilibrium on both sides of the system, as just above, set expected partial demands equal to actual partial demands, and solve the resulting pair of linear equations. In order for this solution to involve positive partial demands, network effects must not be too large:

$$\varphi^m \varphi^c < \frac{(2B^m + \beta^m)(2B^c + \beta^c)}{(B^m + \beta^m)(B^c + \beta^c)}. \tag{S3}$$

Prices are obtained by inverting the two partial demand functions, substituting actual for expected partial demands.

If there are N^a acquirers, with linear demands as above and net unit costs equal to (C^a+T), and N^i issuers, with linear demands as above and net unit costs equal to (C^i-T), the fulfilled expectations equilibrium is as follows:

(S4a) $$Q^m = \frac{(d^m - b^m T)(B^m + \beta^m)}{Z}, Q^c = \frac{(d^c + b^c T)(B^c + \beta^c)}{Z}; \text{ and}$$

(S4b) $$P^a - C^a - T = \frac{d^m - b^m T}{Z},\ P^i - C^i + T = \frac{d^c + b^c T}{Z};$$

where β^m, β^c, D^m, and D^c are defined by equations (13c) and (13d), above,

(S4c) $$d^m = D^m(2B^c+\beta^c) + \varphi^m D^c(S^c+\beta^c),\ d^c = D^c(2B^m+\beta^m) + \varphi^c D^m(S^m+\beta^m),$$

(S4d) $$b^m = B^m(2B^c+\beta^c) - \varphi^m B^c(S^c+\beta^c),\ b^c = B^c(2B^m+\beta^m) - \varphi^c B^m(S^m+\beta^m), \text{ and}$$

(S4e) $$Z = (2B^m+\beta^m)(2B^c+\beta^c) - \varphi^m\varphi^c(S^m+\beta^m)(S^c+\beta^c).$$

Note that condition (S3) is equivalent to $Z > 0$. The necessary and sufficient conditions for equilibrium Q^c to be increasing in T (i.e., $b^c > 0$) and Q^m to be decreasing in T (i.e., $b^m > 0$) also require that network effects not be too large. These two conditions, which are sufficient for $Z > 0$, will both be satisfied for all non-negative β^m and β^c if and only if

(S5) $$\varphi^c < B^c/B^m \text{ and } \varphi^m < B^m/B^c.$$

As in Section IV, total system output is a quadratic in T; it is maximized at

(S6) $$T^{QN} = \frac{1}{2}\left(\frac{d^m}{b^m} - \frac{d^c}{b^c}\right).$$

This, of course, reduces to (14) when $\varphi^i = \varphi^c = 0$. Here, from (S4c) and (S4d), the output-maximizing interchange fee depends on all the cost and demand parameters in the system, including the φs and the βs.

It is somewhat surprising that the bounds on φ^m and φ^c introduced above do not generally suffice to sign the derivatives of T^{QN} with respect to D^m, D^c, B^m, or B^c. (Under bilateral monopoly, T^{QN} is increasing in B^i and decreasing in B^a; if (S5) is satisfied it is increasing in D^a and decreasing in D^i.) On the other hand, it is easy to show that $\partial T^{QN}/\partial\varphi^m > 0$ and $\partial T^{QN}/\partial\varphi^c < 0$. The larger is φ^m, for instance, the greater is the spillover benefit to total output from increasing Q^c by raising T, and this benefit shows up as an increase in Q^m. It is straightforward to show that $\partial T^{QN}/\partial\beta^m < 0$ and $\partial T^{QN}/\partial\beta^c > 0$. All else equal, the more intense is competition on the acquiring side, for instance, the greater the reduction in P^m induced by lowering T; and in this model a reduction in P^m increases Q^c as well as Q^m.

From equations (S4), the system's objective function, V, is proportional to

(S7a) $$V'' = \frac{\omega'}{b^m}\left(d^c + b^c T\right)\left(d^m - b^m T\right)^2 + \frac{1-\omega'}{b^c}\left(d^m - b^m T\right)\left(d^c + b^c T\right)^2, \text{ where}$$

(S7b) $$\omega' = \alpha\frac{2 + [(\beta^c/B^c) - (\varphi^m/B^m)(B^c + \beta^c)]}{2 + \alpha[(\beta^c/B^c) - (\varphi^m/B^m)(B^c + \beta^c)] + (1-\alpha)[(\beta^m/B^m) - (\varphi^c/B^c)(B^m + \beta^m)]}.$$

Comparing equations (17) and (S7), it is straightforward to characterize the privately optimal choice of interchange fee in this model. First, if *$\omega' = 1/2$, private value maximization implies maximization of total system output*. Second, following equation (19), *for fixed T^{QN}, the difference between the privately optimal interchange fee and the output-maximizing interchange fee is a decreasing function of ω'*. (Much as in Section IV, the derivative of total system markup, $[(P^a + P^i) - (C^a + C^i)]$, with respect to T has the sign of $(S^c - b^m)$, which is proportional to the difference between the two bracketed terms in the denominator of (S7b).) This is a relatively weak result, however, since in this model with imperfect competition on both sides of the system, both T^{QN} and ω' depend on the Bs, βs, and φs.

The determination of ω' via (S7b) is sufficiently complex that, almost regardless of system objectives, it would seem hard to guess whether a private value-maximizing interchange fee would be above or below the output-maximizing level. (In the polar case considered at the end of Section IV, as long as the first of conditions (S5) is satisfied, $\beta^m \to \infty$ implies $\omega' \to 0$, leading to a choice of T above the output-maximizing level.) In general, *ω' is increasing in α and φ^c and decreasing in φ^m, and, if conditions (S5) are satisfied, it is increasing in β^c and decreasing in β^m*. It follows that, like the output-maximizing T, *the T that maximizes private value is decreasing in φ^c and increasing in φ^m*.