Supplemental Materials for Julian Wright, "The Determinants of Optimal Interchange Fees in Payment Systems," *The Journal of Industrial Economics* VOLUME (ISSUE), MONTH YEAR, pp. XXX-YYY.

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I Appendix

In this appendix, the equilibrium for merchant acceptance in each industry is derived when merchants compete according to the Hotelling model in Section IV(i) of the corresponding article. The derivation is a straightforward modification of the results of Rochet and Tirole[1] who deal with the case of a single industry where both merchants have the same level of b_S. Their derivations are modified to allow consumers to know which merchants accept cards only a fraction α of the time, and for the fact that the net benefit to consumers from card usage in our model is $b_B - f$ per-transaction, rather than just b_B.

As noted in the corresponding article, consumers will use cards whenever $b_B > f$. Consider first the case where both merchants accept cards in an industry of type b_S. We consider when this will be an equilibrium. When both merchants accept cards the merchants' cost is

$$d + D(f)(m - b_S).$$

Given merchant i's market share

$$s_i = \frac{1}{2} + \frac{p_j - p_i}{2t}$$

the merchant solves

$$\max_{p_i}(p_i - (d + D(f)(m - b_S)))s_i.$$

This implies prices for a merchant in industry of type b_S of

$$p_i = p_j = d + t + D(f)(m - b_S)$$

and equilibrium profits of all merchants that accept cards of

$$\Pi_i = \frac{t}{2}. \tag{1}$$

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[1] Rochet, J-C. and Tirole, J., 2002, 'Cooperation among competitors: Some economics of credit card associations', *Rand Journal of Economics*, 33, pp. 549-570.

Now suppose merchant i deviates by not accepting cards. Consumers of type $b_B < f$ will not want to use cards, so for such consumers the fraction that purchase from firm i is

$$x_i = \frac{1}{2} + \frac{p_j - p_i}{2t}.$$

The same share function applies for those consumers who do want to use cards but do not know whether merchants accept cards or not. For the remaining consumers (a fraction α of those with $b_B > f$), the share that purchase from firm i is

$$x_i = \frac{1}{2} + \frac{p_j - p_i - (b_B - f)}{2t}.$$

Aggregating over all customers,

$$s_i = \frac{1}{2} + \frac{p_j - p_i - \alpha \int_f^\infty (b_B - f)h(b_B)db_B}{2t}$$

and merchant i solves

$$\max_{p_i}(p_i - d)s_i$$

implying prices of

(2)
$$p_i = \frac{1}{2}\left[p_j + t + d - \alpha \int_f^\infty (b_B - f)h(b_B)db_B\right].$$

Similarly, merchant j solves

(3)
$$\max_{p_j}\left[\begin{array}{c}(1 - D(f))(p_j - d)\left(\frac{1}{2} + \frac{p_i - p_j}{2t}\right) \\ +D(f)(p_j - d - m + b_S)\left(\frac{1}{2} + \frac{p_i - p_j + \alpha(\beta_B(f) - f)}{2t}\right)\end{array}\right]$$

which implies

(4)
$$p_j = \frac{1}{2}\left[p_i + t + d + \alpha \int_f^\infty (b_B - f)h(b_B)db_B + D(f)(m - b_S)\right].$$

Solving (2) and (4) simultaneously implies

(5)
$$p_i = t + d + \frac{1}{3}D(f)(m - b_S - \alpha(\beta_B(f) - f))$$

and

(6)
$$p_j = t + d + \frac{1}{3}D(f)(2(m - b_S) + \alpha(\beta_B(f) - f))$$

Substituting (5) into firm i's profit function implies

(7)
$$\Pi_i = \frac{t}{2}\left[1 - D(f)\frac{\alpha(\beta_B(f) - f) + b_S - m}{3t}\right]^2.$$

Comparing (7) with (1), it is clear that merchant i will want to accept cards if

$$b_S \geq m - \alpha(\beta_B(f) - f) \equiv b_S^m$$

which verifies the result in (14) in the corresponding article.

Next consider the possible equilibrium where both firms reject cards. In this case prices are trivially found by

$$\max_{p_j}(p_j - d)\left(\frac{1}{2} + \frac{p_i - p_j}{2t}\right)$$

and so

(8)
$$\Pi_j = \frac{t}{2}.$$

If firm j deviates and accepts cards, while firm i still rejects cards, prices will be given by (5) and (6). Substituting these prices into firm j's profit given by (3) implies

(9)
$$\Pi_j = \frac{t}{2}\left[1 - D(f)\frac{m - b_S - \alpha\left(\beta_B(f) - f\right)}{3t}\right]^2$$
$$-\frac{1}{2t}(m - b_S)D(f)(1 - D(f))\alpha\left(\beta_B(f) - f\right).$$

In an industry of type $b_S \leq b_S^m = m - \alpha\left(\beta_B(f) - f\right)$, merchant j will not want to deviate and accept cards as (9) is strictly less than (8). In this case there is a unique equilibrium where both merchants reject cards. For some $b_S > b_S^m$, but sufficiently close to b_S^m, it will also be an equilibrium for both merchant to reject cards. Following Rochet and Tirole, where there are multiple equilibria (such that merchant profits are identical across the equilibrium) we assume merchants or the card association (which prefers the equilibrium with more merchant acceptance), are able to pick the equilibrium where both merchants accept cards.

The final possibility to consider is of a hybrid equilibrium where merchant j accepts cards while merchant i does not. For this to be an equilibrium requires that the profit in (7) be at least as high as that in (1) and that the profit in (9) be at least as high as that in (8). Provided t is not too small, it is easy to show that if $b_S \leq m - \alpha\left(\beta_B(f) - f\right)$ then $\Pi_j < t/2$ so firm j will not want to accept cards, while if $b_S > m - \alpha\left(\beta_B(f) - f\right)$ then $\Pi_i < t/2$ so firm i will not want to reject cards. Thus, there is no hybrid equilibrium where one merchant accepts and one rejects cards, provided t is not too small.