APPENDIX 1: PROOF OF PROPOSITION 3.1.

(i) We know from Proposition 1 that for any set of firms in the market, profit decreases with the quality-cost margin. We now show that entry (of any additional firm) decreases profits of the others in the market. We know first that some firm *k*'s market share (or else the outside option's share) must decrease following entry (since the entrant is guaranteed a positive share). By the first-order condition (3) firm *k*'s price also falls. Now suppose some other firm *r*'s share rose; so too would *r*'s price (by (3)). But then the price change would imply that *r* is relatively less attractive compared to *k* so that the ratio P_r/P_k should fall, contradicting the share conditions just given. We conclude that all shares must fall; from (3), so too do prices, and hence so do gross profits. Therefore, since potential firms are valued in terms of decreasing quality-cost, there will be a unique cut-off point such that all firms above the cut-off cover their fixed costs and all firms below the cut-off point rationally anticipate they will not be able to cover those costs should they enter. (This argument suggests a simple algorithm for determining how many firms enter: add firms until the $(n+1)^{th}$ firm cannot cover its costs.)

(ii) From Proposition 1, the lowest-ranked firm earns less than all others. From the argument of the previous paragraph, a new entrant (at the bottom of the scale) reduces the profits of all other firms. Hence, an $(n+1)^{th}$ entrant expects a mark-up and a profit less than that of the n^{th} firm at an *n*-firm price equilibrium. Moreover, since the market share of the $(n+1)^{th}$ firm is less than $1/(n+1)$, by Proposition 1, its net revenue goes to zero as *n* goes to infinity. For $K > 0$, an equilibrium therefore exists with a finite number of firms.

APPENDIX 2: PROOF OF PROPOSITION 3.2.

Assume that some good *i* is not produced, but a good $j > i$ with a strictly lower quality-cost is produced. We show that the profit of the firm producing *j* rises if it shifts production to *i*. Let a tilde denote equilibrium values after the shift. Then we claim that $\tilde{p}_i^* - c_i > p_j^* - c_j$ from (4). From the *f.o.c.* (3), this is equivalent to $\tilde{P}_i^* > P_j^*$. Suppose this were not true, i.e.

$$\tilde{P}_i^* \le P_j^*. \qquad \text{A1}$$

Then $\tilde{p}_i^* - c_i \le p_j^* - c_j$ by (3) so that

$$q_i - \tilde{p}_i^* > q_j - p_j^* \qquad \text{A2}$$

(since by hypothesis, $q_i - c_i > q_j - c_j$). Now, since $\tilde{P}_i^* \le P_j^*$, there must be some firm *k* for which $\tilde{P}_k^* \ge P_k^*$. From Firm *k*'s *f.o.c.*, $\tilde{p}_k^* \ge p_k^*$, and so

$$q_k - \tilde{p}_k^* \le q_k - p_k^*. \qquad \text{A3}$$

(A2) and (A3) imply that

$$\frac{\tilde{P}_k^*}{\tilde{P}_i^*} = \exp\left[\frac{(q_k - \tilde{p}_k) - (q_i - \tilde{p}_i^*)}{\mu}\right] < \frac{P_k^*}{P_j^*} = \exp\left[\frac{(q_k - p_k^*) - (q_j - p_j^*)}{\mu}\right],$$

contradicting (A1) and therefore that $\tilde{P}_k^* \ge P_k^*$. Q.E.D.

APPENDIX 3 : THE OVER-ENTRY RESULT WITH NO OUTSIDE GOOD

The welfare function associated to the logit model (2) has the following form (see e.g. McFadden, 1981, and Anderson et al., 1992, for a discussion):

$$W = \mu \ln\left\{\sum_{k=1}^{n} \exp\left[(q_k - c_k)/\mu\right]\right\} - nK.$$

Clearly, the incremental social value of an s^{th} firm is

$$W(s) - W(s-1) = \mu \ln\left\{\frac{D_{s-1} + \exp\left[(q_s - c_s)/\mu\right]}{D_{s-1}}\right\} - K,$$

where $D_{s-1} = \sum_{k=1}^{s-1} \exp\left[(q_k - c_k)/\mu\right]$. The logarithm term is less than $\frac{\exp[(q_s - c_s)/\mu]}{D_{s-1}}$ (and approximately equal to this when it is small). Hence the welfare gain from the s^{th} firm is less than

$$\mu \frac{\exp[(q_s - c_s)/\mu]}{D_{s-1}} - K. \qquad \text{A4}$$

We now show that the profit of the s^{th} firm is greater than this value, and thus that firms will enter the market even when their net social worth given by (A4) is negative (leading to over-entry). Using (8), this amounts to showing that

$$\exp[(-c_s)/\mu]\sum_{k=1}^{s-1}\exp[(q_k - p_k^*)/\mu] < \exp[(-p_s^*)/\mu]\sum_{k=1}^{s-1}\exp[(q_k - c_k)/\mu].$$

This inequality holds since $q_i - p_i^* - c_s < q_i - p_s^* - c_i$, for all $i \neq s$, by Proposition 1. The discussion above is summarised by the following result:

For the logit model (2) *with asymmetric costs and qualities, there is excessive entry of firms in the market equilibrium.*

When firms are symmetric (quality-cost is the same for all firms), the number of firms is approximately the social optimum level (the extent of over-entry for the logit is just one firm: see Anderson, de Palma, and Thisse, 1992). With asymmetric qualities and costs, the over-entry problem can be *much more severe*. To illustrate the possible extent of the problem, suppose that marginal costs are zero and $\mu = 1$. There are 20 products which have high quality $(q_1 = ... = q_{20} = Q_H = 4)$ and 20 products with low quality ($q_{21} = ... = q_{40} = Q_L = 1$). Let $K = 0.0025$. Then it can be shown that the optimum involves only the 20 high-quality firms, but the equilibrium has all 40 firms entering. footnote