Supplemental Materials for Begoña Garcia Mariñoso, "Technological incompatibility, endogenous switching costs and lock in" ***The Journal of Industrial Economics***

Appendix for Section V: Technological Choice.

a. Ex-ante profit comparisons: Comparing ex-ante profits with incompatibility (5) and compatibility (6):

1. For $c_x \leq 3L$, $p_{EA}^{C} - p_{EA}^{I} = d \cdot c_x / 3 \cdot (6L - c_x)/6 - F$

2. For $c_x \geq 3L$, $p_{EA}^{C} - p_{EA}^{I} = d \cdot L^2 / 2 - F$

b. Welfare Comparisons: At t=1 welfare is the same under all technological regimes: $W_1 = Lw - cL - L^2/4$. Second period welfare determines which regime is best. The expressions for second period surplus and welfare under incompatibility and compatibility are:

- For incompatibility and $c_x \leq 3L$:

$$S_2^I = \int_0^{L/2+c_x/6} v - x - (L + c_y + c_x/3)\P x + \int_{L/2+c_x/6}^{L} v - (L - x) - (L + c_y + 2/3\, c_x)\P x =$$

(13)
$$= L\,v - c_y\,L - 5/4\,L^2 + c_x^2/36 - L\,c_x/2$$
$$W_2^I = L\,v - c_y\,L - L^2/4 + 5\,c_x^2/36 - L\,c_x/2$$

- For incompatibility and $c_x > 3L$:

(14)
$$S_2^I = \int_0^L v - x - (c - L)\P x = Lv - cL + L^2/2$$
$$W_2^I = Lv - c_y L - L^2/2$$

- For compatibility:

(15)
$$S_2^C = \int_0^{L/2} v - x - (c_y + L)\P\, x + \int_{L/2}^{L} v - (L - x) - (c_y + L)\,\P\, x = L\,v - c_y L - 5\,L^2/4$$
$$W_2^C = L\,v - c_y\,L - L^2/4 - 2F$$

Comparing (13) with (15) and (14) with (15):

1. For $c_x \leq 3L$, $W_{EA}^C - W_{EA}^I = dc_x/36 * (18\,L - 5\,c_x) - 2F$
2. For $c_x \geq 3L$, $W_{EA}^C - W_{EA}^I = dL^2/4 - 2F$

c. Proof of Proposition 4

- For $c_x \leq 3L$:

$$If\ F \leq d \cdot c_x \cdot (18L - 5 \cdot c_x)/72,\ then:\ W_{EA}^{C} \geq W_{EA}^{I}\ and\ p_{EA}^{C} \geq p_{EA}^{I},$$
$$if\ \ d \cdot c_x \cdot (18L - 5 \cdot c_x)/72 \leq F \leq d \cdot c_x \cdot (6L - c_x)/18\ \ then:\ W_{EA}^{C} \leq W_{EA}^{I}\ and\ p_{EA}^{C} \geq p_{EA}^{I},$$
$$and\ \ finally,\ if\ \ F \geq d \cdot c_x(6L - c_x)/18\ \ then:\ W_{EA}^{C} \leq W_{EA}^{I}\ and\ p_{EA}^{C} \leq p_{EA}^{I}.$$

- For $c_x \geq 3L$:

$$If\ \ F \leq d \cdot L^2/4, then\ W_{EA}^{C} \geq W_{EA}^{I}\ and\ p_{EA}^{C} \geq p_{EA}^{I},$$
$$if\ \ d \cdot L^2/4 \leq F \leq d \cdot L^2/2, then\ W_{EA}^{C} \leq W_{EA}^{I}\ and\ p_{EA}^{C} \geq p_{EA}^{I}$$
$$and\ if\ \ F \geq d \cdot L^2/2,\ then W_{EA}^{C} \leq W_{EA}^{I}\ and\ p_{EA}^{C} \leq p_{EA}^{I}.$$

Proposition 4 follows.

Appendix for Section VI: Brand Loyalty.

a. Second Period

a.1- Incompatibility: With incompatibility, indifferent consumers and profits are as reported in (1) and (2) in section 3. Hence, Lemma 1 and Proposition 2 hold.

a.2- Compatibility: Indifferent consumers in segment A are as reported brand loyalty section posted in the web: I_{AM}^{A}, I_{BM}^{A} and $I_{AB}^{A} = (P_B - P_{AY} + L)/2$ (indifferent between $x_A y_A$ and $x_B y_B$). Similarly, for segment B: $I_{BM}^{B} = P_{BY} - P_{AY} + L/2$, $I_{AM}^{B} = (L/2 + P_{BX})$ and $I_{AB}^{B} = (P_{BY} - P_A + L)/2$.

Two regimes can arise: $I_{AM}^{A} \geq I_{BM}^{A}$, implying that no consumer in segment A is willing to mix and match, and $I_{AM}^{A} \leq I_{BM}^{A}$, where some consumers buy $x_A y_B$. Note that in symmetric equilibrium if 0 P_{JX} >0, there is mix and match in both segments.

(i) Proof of Lemma 5

Firm A's profit with mix and match is as reported in (12). Since $I_{AM}^{A} = I_{BM}^{B}$, (12) can be expressed as:

(16) $$p_{2A} = (P_{AY} - c_y)(I_{AM}^{A}) + (P_{AX} - c_x)(s_B / L.I_{AM}^{B})$$

Standard optimisation of (16) yields reaction functions for Firm A: $P_{BY} - 2P_{AY} + L/2 + c_y = 0$ and $L/2 - 2P_{AX} + c_x = 0$. By finding their intersection with reaction functions for firm B the prices market shares and profits in Lemma A result. To check that these prices are profit maximizing one must check that at given rival's prices no firm wants to set prices such that the regime reverts to a non- mix and match regime for some segment.

- Taking $P_{BY} = L/2 + c_y \ and\ P_{BX} = 1/2(c_x + L/2)$ as given, indifferent consumers are:

$$I^A_{AB} = 7L/8 + c_y/2 + c_x/4 - P_{AY}/2, X^A_{AM} = L + c_y - P_{AY}, and$$
$$I^A_{BM} = 3L/4 + c_x/2$$
$$I^B_{AB} = 3L/4 + c_y/2 - P_A/2, X^B_{AM} = L/2 - P_{AX}, and$$
$$I^B_{BM} = L + c_y - P_{AY}$$

There is mix and match in segment A if $0 \le P_{AY} - L/4 - c_y + c_x/2$ (17)

and no mix and match in segment A if $0 \ge P_{AY} - L/4 - c_y + c_x/2$ (18)

There is mix and match in segment B if $0 \le L/2 + c_y + P_{AX} - P_{AY}$ (19)

and no mix and match in segment B if $0 \ge L/2 + c_y + P_{AX} - P_{AY}$ (20)

Firm A can deviate from the mix and match regime in three ways: By setting prices such that: there is only mix and match in segment B (deviation 1), there is only mix and match in segment A (deviation 2), or there is no mix and match in either segment (deviation 3). I prove that these three deviations are not profitable for firm A.

Deviation 1: The profit function for firm A is:

$$p_{D1} = (P_{AY} - c_y)(s_A/L \cdot I^A_{AB} + s_B/L \cdot I^B_{BM}) + (P_{AX} - c_x)s_B/L \cdot I^B_{BM}$$

Then, the optimisation problem for firm A is to:

$$Max_{P_{AY}, P_{AX}}\, p_A \text{ such that}:$$

(18), (19) and $P_{AY} \ge 0, P_{AX} \ge 0.$

and the Kuhn Tucker conditions to be satisfied by the solution to this programme are:

$$\P p_A / \P P_{AY} - l_1 - l_2 = 0$$
$$\P p_A / \P P_{AX} + l_2 = 0$$
$$l_1 \cdot (L/4 + c_y - c_x/2 - P_{AY}) = 0$$
$$l_2 \cdot (L/2 + c_y - P_{AY} + P_{AX}) = 0$$
$$P_{AX} \ge 0, P_{AY} \ge 0, l_1 \ge 0, l_2 \ge 0$$

where l_1 is the multiplier associated with (18) and l_2 is the multiplier associated with (19).

The solution is:

$$l_1 = s_A / L \cdot (3c_x / 4 + 5L / 8) + s_B / L \cdot (L / 2 + c_x)$$
$$l_2 = 0$$
$$P_{AY} = L / 4 + c_y - c_x / 2$$
$$P_{AX} = L / 4 + c_x / 2$$

and the value of profit at this solution is:

$$p^*_{D1} = (L / 4 - c_x / 2) \cdot (s_A / L \cdot (3L / 4 + c_x / 2 - c_y) + s_B)$$

This value is smaller than the value of profits at the candidate equilibrium in Lemma 5.

Deviation 2: The profit function for firm A is:

$$p_{D2} = (P_{AY} - c_y)(s_A / L \cdot I^A_{AM}) + (P_A - c_x - c_y) s_B / L \cdot I^B_{AB}$$

Then, the optimisation problem for firm A is to:

$$Max_{P_{AY}, P_{2X}} \, p_A \;\; such \;\; that:$$
$$(17), (20) \text{ and } P_{AY} \geq 0, P_{AX} \geq 0.$$

and the Kuhn Tucker conditions to be satisfied by the solution to this programme are:

$$\P p_A / \P P_{AY} + l_1 + 2 l_2 = 0$$
$$\P p_A / \P P_A - l_2 = 0$$
$$l_1 (-L / 4 - c_y + c_x / 2 + P_{AY}) = 0$$
$$l_2 (-L / 2 - c_y + 2P_{AY} - P_A) = 0$$
$$P_A \geq 0, P_{AY} \geq 0, \; l_1 \geq 0, \; l_2 \geq 0$$

where l_1 is the multiplier associated with (17) and l_2 is the multiplier associated with (20).

The solution is:

$$l_1 = 0$$
$$l_2 = s_B / L \cdot (c_y + c_x / 2 + 3L / 4 - 1/(2s_B + s_A) \cdot [s_B \cdot (2c_y + c_x + 3L / 2) + s_A \cdot (c_y + L / 2)])$$
$$P_{AY} = L / 4 + c_y / 2 + 1 / 2 \cdot [1/(2s_B + s_A) \cdot (s_B \cdot (2c_y + c_x + 3L / 2) + s_A \cdot (c_y + L / 2))]$$
$$P_A = 1/(2s_B + s_A) \cdot [s_B \cdot (2c_y + c_x + 3L / 2) + s_A \cdot (c_y + L / 2)]$$

and the value of profit at this solution is:

$$p^*_{D2} = \frac{1}{2(L + s_B)^2} \bullet (L^2 + s_B(L / 2 - c_x)) \bullet (L^3 / 2 + 3L^2 \, s_B / 4 + L \, s_B^2 / 4 - 3 \, s_B^2 \, c_x \, L / 2 + s_B \, c_x \, L^2 / 2)$$

This value is smaller than the value of profits at the candidate equilibrium in Lemma 5.

Deviation 3: The profit function for firm A is:

$$p_{D3} = (P_{AY} - c_y) \cdot s_A / L \cdot I^A_{AB} + (P_A - c_x - c_y) \cdot s_B / L \cdot I^B_{AB}$$

Then, the optimisation problem for firm A is to:

$Max_{P_{2AY}, P_{2X}} p_A$ such that :

$(18), (20)$ and $P_{AY} \geq 0, P_{AX} \geq 0$

and the Kuhn Tucker conditions to be satisfied by the solution to this programme are:

$$¶ p_A / ¶ P_{AY} - l_1 + 2 l_2 = 0$$
$$¶ p_A / ¶ P_A - l_2 = 0$$
$$l_1 \cdot (L/4 + c_y - c_x/2 - P_{AY}) = 0$$
$$l_2 \cdot (-L/2 - c_y + 2P_{AY} - P_A) = 0$$
$$P_{AX} \geq 0, P_{AY} \geq 0, l_1 \geq 0, l_2 \geq 0$$

where l_1 is the multiplier associated with (18) and l_2 is the multiplier associated with (20).

The solution is:

$$l_1 = 3 s_B \cdot (c_x + L/2)/L + s_A (5L/8 + 3c_x/4)$$
$$l_2 = 3 s_B \cdot (c_x + L/2)/(2L)$$
$$P_{AY} = L/4 + c_y - c_x/2$$
$$P_A = c_y - c_x$$

and the value of profit at this solution is:

$$p^*_{D3} = (3L/4 + c_x/2)(s_A/4 - (c_x s_A)/(2L) - 2 c_x s_B / L)$$

This value is smaller than the value of profits at the candidate equilibrium in Lemma 5.

b. First Stage: At t=1 the indifferent consumer is: $I = Q_B - Q_A + L/2$. Firm A (and equivalently, firm B) chooses Q_A to maximize ex-ante profits: $(Q_A - c_x) I + d \cdot p^S_{2A}$ with S={C, I}.

- Incompatibility: Standard optimisation yields prices for firm *J*:

(i) For $c_x \leq 3L$, $Q_J = L/2 + c_x - 2 d c_x / 3$ and $s_J = L/2$ and

(ii) For $c_x \geq 3L$, $Q_J = L/2 + c_x - d(c_x - L)$ and $s_J = L/2$.

- Compatibility: Standard optimisation yields prices for firm *J*:

(i) For $c_x \leq L/2$, $Q_J = L/2 + c_x + d/L\,(L/4 - c_x/2)^2$ and $s_J = L/2$

Substituting these prices and market shares in the relevant profit functions, I obtain the value for profits reported in Table 1.

c. Lemma B: Comparing ex-ante profits for compatibility and incompatibility when $c_x \leq L/2$ and *F=0*: $p^I_{EA} - p^C_{EA} = d/72(-14 c_x^2 - 6 c_x L + 27 L^2 / 2)$>0. Lemma B follows.

d. Welfare (Proposition 7): The welfare comparison only depends on welfare levels at t=2. For $c_x \leq L/2$ expression (ß) gives the levels of welfare for incompatibility. With compatibility, if *F=0*, consumer surplus and welfare is:

$$S_2^C = \int_0^{L/2} (v - P_{AY} - x) ¶ x + \int_{L/2}^{3L/4+c_x/2} (v - P_{BY} - L/2) ¶ x + \int_{3L/4+c_x/2}^{L} (v - P_B - (L-x)) ¶ x =$$

(21) $$= (32 L v - 32 c_y L + 4 c_x^2 - 4 c_x L - 27 L^2)/32$$

$$W_2^C = S_2^C + p_2^C = (32 L v - 32 c_y L + 12 c_x^2 - 12 c_x L - 9L^2)/32$$

Comparing (21) and (13): $W_{EA}^C - W_{EA}^I = d(17 c_x^2 / 72 + c_x L / 8 - L^2 / 32)$. Proposition 7 follows.